Passive Decomposition and Control of Nonholonomic Mechanical Systems

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***Abstract*—We propose nonholonomic passive decomposition, which enables us to decompose the Lagrange–D'Alembert dynamics of multiple (or a single) nonholonomic mechanical systems with a formation-specifying (holonomic) map h into 1) shape system, describing the dynamics of $h(q)$ (i.e., formation aspect), where $q \in \Re^n$ is the systems' configuration; 2) locked system, describing the systems' motion on the level set of h with the formation aspect $h(q)$ being fixed (i.e., maneuver aspect); 3) quotient system, whose nonzero motion perturbs both the formation and maneuver aspects simultaneously; and 4) energetically conservative inertia-induced coupling among them. All the locked, shape, and quotient systems individually inherit Lagrangian dynamics-like structure and passivity, which facilitates their control design/analysis. Canceling out the coupling, regulating the quotient system, and controlling the locked and shape systems individually, we can drive the formation and maneuver aspects simultaneously and separately. Notions of formation/maneuver decoupled controllability are introduced to address limitations imposed by the nonholonomic constraint, along with passivity-based formation/maneuver control design examples. Numerical simulations are performed to illustrate the theory. Extension to kinematic nonholonomic systems is also presented.**

***Index Terms*—Decomposition, geometry, multirobot formation control, nonholonomic mechanical systems, passivity.**

I. Introduction

LET US consider a nonholonomic mechanical system with its configuration $q \in \Re^n$, whose evolution is governed by the Lagrange–D'Alembert equation (with kinetic energy as Lagrangian) and nonholonomic Pfaffian constraint [1]. This description is also applicable to a team of multiple nonholonomic mechanical systems as well, if we cast their individual dynamics into their product manifold [2], [3]. Either for a single system or for a team of multiple of them, in many practical applications, we often want to achieve some sort of motion coordination (or formation) requirements, e.g., internal degrees-of-freedom (DOFs) coordination within a single (redundant) robot to avoid obstacles, while maintaining the end-effect position stationary; or interrobot formation shape control to hold a deformable object in *fixtureless* multirobot cooperative manipulation. For such applications, it is also usually possible to describe such motion coordination or formation requirements by the mapped point $h(q)$ of a certain (holonomic) map h defined on the systems' configuration space (e.g., $h(q) =$ end-effector position of the redundant robot, or $h(q_1, q_2) = q_1 - q_2$ with $q_1, q_2 \in \Re^3$ being the position of two point masses). Let us call such a map h formation map.

In this paper, extending the standard passive decomposition [2]–[4], we reveal a fundamental property of nonholonomic mechanical systems in this setting. More specifically, we show that, for a general nonholonomic mechanical system with a (submersion) formation map h, we can decompose its Lagrange–D'Alembert dynamics into 1) shape system, representing the *formation aspect*, i.e., how $h(q)$ is changing; 2) locked system, describing the *maneuver aspect*, i.e., the system's behavior on the level set of h with the formation aspect $h(q)$ being locked; 3) quotient system, whose nonzero motion perturbs both the formation and maneuver aspects simultaneously; and 4) inertia-induced conservative (i.e., skew symmetric) couplings among the locked, shape, and quotient systems, which are functions of (usually accessible) q and $\dot{q}$, and quadratic in $\dot{q}$. All the locked, shape, and quotient systems individually inherit Lagrangian dynamics-like structure and passivity, which greatly facilitate their control design/analysis (e.g., passivity-based control techniques readily applicable). For this reason, following [2]–[4], we name our decomposition *nonholonomic passive decomposition (NPD)*. We also show that, for some special cases, the quotient system vanishes, which results in the locked-shape only decomposition similar to the case of standard passive decomposition.

Canceling out the inertia-induced coupling and regulating the quotient system (if not vanished), we can then achieve formation-maneuver decoupling; also, controlling the (decoupled) locked and shaped systems individually, we can drive the maneuver and formation aspects simultaneously and separately. These formation-maneuver decoupling and their simultaneous/separate control are desired and often necessary in many applications. For instance, consider *fixtureless* multirobot cooperative manipulation, with $h(q)$ describing the grasping shape among the robots. Then, we would want to maintain a certain desired cooperative grasping shape (i.e., formation aspect) and drive the grasped object according to a certain task objective (i.e., maneuver aspect) at the same time. We would also want these two behaviors to be decoupled from each other, since if they are not, driving the grasped object may perturb the cooperative grasping, thereby resulting in (potentially dangerous) dropping of the object.

Even with this formation-maneuver decoupling and their separate control, the system's nonholonomic constraint may impose

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fundamental restrictions on the achievable formation and maneuver behaviors. For this, we define and elucidate new notions of formation and maneuver decoupled controllability (or d-controllability), satisfaction of which then guarantees that we can control one aspect without perturbing the other; also, doing so is not hindered by the nonholonomic constraint (in the sense of Chow [1]). We also provide passivity-based control design examples that can achieve certain desired formation/maneuver aspects simultaneously and separately by utilizing inherited Lagrangian-like structure and passivity of the decomposed systems. We also present extension of these results to kinematic nonholonomic systems, which, yet, like any other kinematic result, can neither address "dynamic" phenomena (e.g., inertia, force, power, and energy) nor allow us to reveal how to exploit open-loop dynamics of the (dynamic) nonholonomic mechanical systems.

Many strong results have been reported for mechanical systems, if they are only with nonholonomic constraint (e.g., [6]–[8]) or only with holonomic constraint (i.e., $h(q) = c$ is somehow enforced $\forall t \geq 0$ with a constant c [9], [10]).[1] However, results similar to our NPD are very rare. This rareness, we believe, is due to the complex geometry arising from the coexistence of the holonomic formation map h and the nonholonomic constraint. To our knowledge, only the results similar to our NPD are: 1) motion feasibility of [11], which, yet, only specifies when the locked system is not null for kinematic nonholonomic systems, thus leaving many important issues unanswered (e.g., formation/quotient aspects and controllability) as well as unable to address such important dynamic phenomena as inertial effect and external force; and 2) standard passive decomposition [2]–[5], which, although powerful for unconstrained mechanical systems, is not so suitable for nonholonomic systems, as shown in Section II-C. In this sense, our results of NPD here may be thought of as an extension of the standard passive decomposition to nonholonomic systems; and a generalization of the results of [11] to the (dynamic and kinematic) nonholonomic systems.

The rest of this paper is organized as follows. Some preliminary materials, including a brief summary of standard passive decomposition and its limitations for nonholonomic systems, are given in Section II. The main result of this paper, NPD, is presented and its geometry/energetics detailed in Section III. The notions of formation/maneuver d-controllability are introduced and control design examples provided in Section IV. The extension to kinematic nonholonomic systems is presented in Section V. Some illustrative examples and simulation results are discussed in Section VI. Summary and remarks on future research in Section VII conclude the paper. Some portions of this paper have been presented in [12, Sec. III-A] and [13, Secs. III-B and IV].

[1] This case of holonomic constraint $h(q) = c$ is often easier to deal with than the case of *controlling* $h(q)$ as we attempt here since, with $h(q) = c$, system dynamics is usually reduced to a similar dynamics on the level set $\mathcal{H}_c$ (8) with fewer-DOFs [3], [9].

II. Preliminary

A. Nonholonomic Mechanical Systems

Let us start with the dynamics of nonholonomic mechanical systems, which consists of 1) nonholonomic Pfaffian constraint

$$A(q)\dot{q} = 0 \tag{1}$$

and 2) Lagrange–D'Alembert equation of motion

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + A^T(q)\lambda = \tau + f \tag{2}$$

where $q, \dot{q}, \tau, f \in \Re^n$ are the configuration, velocity, control, and external force, $M, C \in \Re^{n\times n}$ are the inertia and Coriolis matrices with $\dot{M} - 2C$ being skew symmetric [14], $A(q) \in \Re^{p\times n}$ $(p \leq n)$ defines the nonholonomic constraint, and $A^T(q)\lambda$ is the constraint force, whose magnitude is specified by the Lagrange multiplier $\lambda \in \Re^p$. Here, we (locally) identify the system's configuration space $\mathcal{M}$ by $\Re^n$ (i.e., $\mathcal{M} \approx \Re^n$). We also assume that the nonholonomic constraint (1) is smooth and regular (i.e., $\text{rank}A(q) = p$ for all q). This modeling (1) and (2) is applicable to multiple nonholonomic mechanical systems as well, by combination of their individual dynamics and constraints into their product configuration space [2], [3].

Using the constraint (1) and the inertia metric $M(q)$, we can then generate four spaces at each q: 1) *constrained codistribution* $\mathcal{C}^\perp$, which is the row space of $A(q)$ determining the space of constraint forces; 2) *unconstrained distribution* $\mathcal{D}^\top$, which is the kernel of $A(q)$ specifying the direction of $\dot{q}$ permitted by the constraint (1); 3) *constrained distribution* $\mathcal{D}^\perp$, which is the orthogonal complement of $\mathcal{D}^\top$ w.r.t. the $M(q)$-metric; and 4) *unconstrained codistribution* $\mathcal{C}^\top$, which annihilates $\mathcal{D}^\perp$. Note that $\mathcal{C}^\perp$ also annihilates $\mathcal{D}^\top$. Here, the first two are purely kinematic [i.e., only dependent on the constraint (1)], thus, easy to compute, while the last two are inertia-dependent.

Then, at each q, the tangent space (i.e., velocity space: $T_q\mathcal{M}$) and the cotangent space (i.e., force space: $T_q^*\mathcal{M}$), respectively, split s.t.

$$T_q\mathcal{M} = \mathcal{D}^\top \oplus \mathcal{D}^\perp \quad \text{and} \quad T_q^*\mathcal{M} = \mathcal{C}^\top \oplus \mathcal{C}^\perp \tag{3}$$

where $\oplus$ is the direct sum, and $\dot{q}$ and τ can be written as follows:

$$\dot{q} = \underbrace{[\mathcal{D}_\top \quad \mathcal{D}_\perp]}_{=:\mathcal{D}(q)} \begin{pmatrix} \nu \\ \xi \end{pmatrix}, \tau = \underbrace{[\mathcal{C}_\top^T \quad \mathcal{C}_\perp^T]}_{=:\mathcal{C}^T(q)} \begin{pmatrix} u \\ u_\xi \end{pmatrix} \tag{4}$$

where $\mathcal{D}_\top \in \Re^{n\times(n-p)}$, $\mathcal{D}_\perp \in \Re^{n\times p}$, $\mathcal{C}_\top \in \Re^{(n-p)\times n}$, and $\mathcal{C}_\perp \in \Re^{p\times n}$ are the matrices identifying their respective spaces. Similarly, we can write $f = \mathcal{C}^T(q)[\delta;\delta_\xi]$, where ";" is the "append" operator defined s.t., for $x \in \Re^{n_x}$, $y \in \Re^{n_y}$, $[x;y] := [x^T, y^T]^T \in \Re^{n_x+n_y}$. Since $\mathcal{D}_\perp$ describes the direction of velocity violating the constraint (1), $\xi = 0$. Note also that the control/force in $\mathcal{C}_\top$ direction (i.e., u, δ) is fully effective, while those in $\mathcal{C}_\perp$ (i.e., u_ξ, δ_ξ) are completely absorbed by the constraint force.

From our construction, $\mathcal{C}_\perp \mathcal{D}_\top = 0$, and $\mathcal{C}_\top \mathcal{D}_\perp = 0$. Also, to have the following power preservation s.t.:

$$\text{power}(t) := (\tau + f)^T \dot{q} = (u + \delta)^T \nu + (u_\xi + \delta_\xi)^T \xi \tag{5}$$

we enforce $\mathcal{C}_\top \mathcal{D}_\top = I$ and $\mathcal{C}_\perp \mathcal{D}_\perp = I$. This can be achieved by simply setting $\mathcal{C} = \mathcal{D}^{-1}$. Here, since $\xi = 0$, the last term in (5) is zero. Using (1), (4), and the skew symmetricity of $\dot{M} - 2C$, we can then show the (energetic) passivity of the (original and projected) nonholonomic mechanical system (1) and (2) with the power (5) as the supply rate and the kinetic energy $\kappa(t) := \dot{q}^T M(q)\dot{q}/2$ as the storage function, s.t.

$$\int_0^T (\tau + f)^T \dot{q} dt = \int_0^T (u + \delta)^T \nu dt = \kappa(T) - \kappa(0) \quad (6)$$

for all $T \geq 0$. See also [12].

B. Formation Map

In this paper, we suppose that the motion coordination or formation requirements for (1) and (2) can be represented by the mapped point of a (holonomic) map

$$h : \Re^n \rightarrow \Re^m, \quad m \leq n \quad (7)$$

which, for instance, may be the end-effector position of a redundance robot, if we are interested in keeping this position stationary (i.e., $h(q) = h_d$ for all $t \geq 0$), while avoiding obstacles via the internal motion; or $h(q) := [q_1 - q_2; q_2 - q_3] \in \Re^6$ with $q := [q_1; q_2; q_3] \in \Re^9$ and $q_i \in \Re^3$ being the position of three point masses, if we are interested in achieving a certain formation shape among them with $h(q) \rightarrow h_d$, where for $x, y \in \Re^n$, $x \rightarrow y$ implies $\|x - y\| \rightarrow 0$, with $\|\cdot\|$ being any vector norm. Following the latter application of h, we call this map h *formation map* and its range space $\mathcal{N} \approx \Re^m$ *formation manifold*.

We further assume that this formation map h is a smooth submersion (i.e., its Jacobian is full rank). Then, the level set of h, which is defined by

$$\mathcal{H}_c := \{q \in \Re^n | h(q) = c, c \in \Re^m\} \quad (8)$$

constitutes a $(n - m)$-dimensional smooth submanifold in $\Re^n$ and the collection of them forms a foliation [15]. We may then think of the two motion aspects for the nonholonomic mechanical system (1) and (2): 1) *formation aspect*, i.e., how the mapped point $h(q)$ moves on $\mathcal{N}$ (e.g., interagent formation shape); and 2) *maneuver aspect*, i.e., how the system's motion evolves on the level set $\mathcal{H}_{h(q)}$ (e.g., overall team motion with the interagent formation shape fixed). See Fig. 1.

C. Standard Passive Decomposition

Here, we briefly review the standard passive decomposition and show its limitations for nonholonomic systems. For more details, see [2]–[5] and [16]. Let us start with the observation that, at each q, for the velocity $\dot{q}$ to be parallel to the (current) level set $\mathcal{H}_{h(q)}$, it needs to satisfy

$$\mathcal{L}_{\dot{q}} h = \frac{\partial h}{\partial q} \dot{q} = 0$$

where $\mathcal{L}_{\dot{q}} h$ is the Lie derivative of h along $\dot{q}$. In other words, the kernel of $\partial h/\partial q \in \Re^{m \times n}$ defines the distribution (i.e., subspace of velocity) parallel to $\mathcal{H}_{h(q)}$. Then, similar to Section II-A, using $M(q)$-metric, we can define the following four vector spaces: 1) *normal codistribution* $\Omega^\perp$, which is the row space

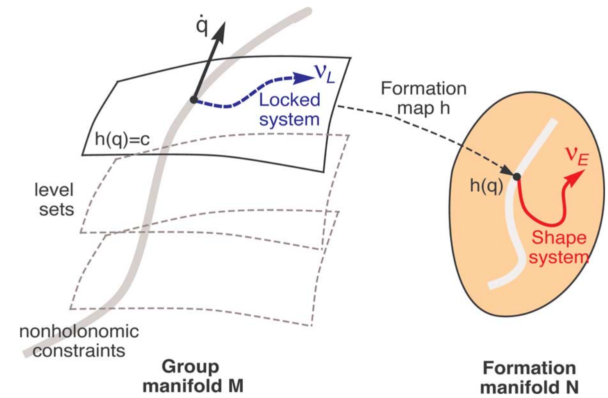

Fig. 1. Geometry of formation map h and level set $\mathcal{H}_c$.

of $\partial h/\partial q$ representing the force directions normal to $\mathcal{H}_{h(q)}$; 2) *parallel distribution* $\Delta^\top$, which is the kernel of $\Omega^\perp$, thus, is parallel to $\mathcal{H}_{h(q)}$ and constitutes the velocity space of the maneuver aspect; 3) *normal distribution* $\Delta^\perp$, which is the orthogonal complement of $\Delta_\top$ w.r.t. the $M(q)$-metric, whose image via h on $\mathcal{N}$ describes the evolution of the formation aspect $h(q)$; and 4) *parallel codistribution* $\Omega^\top$, which annihilates $\Delta^\perp$ and encodes the force directions to affect only the maneuver aspect along $\mathcal{H}_{h(q)}$. Again, the former two are purely kinematic (i.e., dependent only on h), while the latter two are inertia-dependent.

Then, similar to (3) and (4), we have, at each q

$$T_q\mathcal{M} = \Delta^\top \oplus \Delta^\perp, \quad T_q^*\mathcal{M} = \Omega^\top \oplus \Omega^\perp \quad (9)$$

and we can write $\dot{q}$ and τ by (similar also hold for f)

$$\dot{q} = \underbrace{[\Delta_\top \quad \Delta_\perp]}_{=:\Delta(q)} \begin{pmatrix} v_L \\ v_E \end{pmatrix}, \tau = \underbrace{[\Omega_\top^T \quad \Omega_\perp^T]}_{=:\Omega^T(q)} \begin{pmatrix} \tau_L \\ \tau_E \end{pmatrix} \quad (10)$$

where the matrices $\Delta_\top \in \Re^{n\times(n-m)}$, $\Delta_\perp \in \Re^{n\times m}$, $\Omega_\top \in \Re^{(n-m)\times n}$, and $\Omega_\perp \in \Re^{m\times n}$ identify their respective spaces. Similar to (4), we also enforce $\Omega\Delta = I$ (e.g., $\Omega = \Delta^{-1}$).

Many choices are possible for these decomposition matrices. One particularly insightful choice is as follows; define $\Delta_\top$ as *any* $(n - m)$-dimensional kernel space of $\partial h/\partial q$; then, choose $\Omega_\perp := \partial h/\partial q$, $\Delta_\perp := M^{-1}\Omega_\perp^T(\Omega_\perp M^{-1}\Omega_\perp^T)^{-1}$, and $\Omega_\top := (\Delta_\top^T M \Delta_\top)^{-1}\Delta_\top^T M$. We can then show that: 1) $\Delta_\top^T M \Delta_\perp = 0$ (i.e., orthogonal w.r.t. the $M(q)$-metric); 2) $\Omega\Delta = I$ with power preservation/decomposition $(\tau + f)^T \dot{q} = \tau_L^T v_L + \tau_E^T v_E$ similar to (5); and 3) $dh/dt = (\partial h/\partial q)\dot{q} = \Omega_\perp \Delta_\perp v_E = v_E$ (with $\Omega\Delta = I$), i.e., v_E explicitly describes the formation aspect $h(q)$ on $\mathcal{N}$. The results of this paper and their implementation do not depend on this (or any) specific choice for (10). Since (10) is used only for analysis without being implemented, *any* choice can then be assumed for (10).

Using $\Delta_\top^T M \Delta_\perp = 0$, we can then decompose the original dynamics (2) into: with argument omitted for brevity

$$M_L \dot{v}_L + C_L v_L + C_{LE} v_E + \Delta_\top^T A^T \lambda = \tau_L + f_L \quad (11)$$

$$M_E \dot{v}_E + C_E v_E + C_{EL} v_L + \Delta_\perp^T A^T \lambda = \tau_E + f_E \quad (12)$$

where $M_L = \Delta_\top^T M \Delta_\top$, $M_E = \Delta_\perp^T M \Delta_\perp$, and

$$\begin{bmatrix} C_L & C_{LE} \\ C_{EL} & C_E \end{bmatrix} := \Delta^T [M\dot{\Delta} + C\Delta]. \tag{13}$$

Note that the first dynamics (11) is the projection of the original dynamics (2) onto $\Delta^\top$ (i.e., maneuver aspect), while the second (12) onto $\Delta^\perp$ (i.e., formation aspect). We call the dynamics of v_E in (12) *shape system*, which specifies the formation aspect, i.e., the change of $h(q)$ on $\mathcal{N}$ (e.g., with $v_E = dh/dt$), while the dynamics of v_L in (11) *locked system*, which describes the maneuver aspect, i.e., the system's motion within a single level set with the formation aspect $h(q)$ being locked. Here, due to the orthogonality of $\Delta^\top$ and $\Delta^\perp$ w.r.t. the $M(q)$-metric, there is no (usually noncancelable) acceleration coupling between the locked and shape systems. The following Proposition 1 summarizes properties of the standard passive decomposition, for whose proof we refer readers to [3], [4], [16], or the (similar) proof of Theorem 1.

Proposition 1: Consider the decomposed dynamics (11) and (12). Then, we have the following.

1) M_L and M_E are symmetric and positive definite.
2) $\dot{M}_L - 2C_L$ and $\dot{M}_E - 2C_E$ are skew symmetric.
3) $C_{LE} = -C_{EL}^T$.
4) Kinetic energy and power are decomposed s.t.

$$\kappa(t) = \kappa_L(t) + \kappa_E(t), \quad \tau^T \dot{q} = \tau_L^T v_L + \tau_E^T v_E$$

where $\kappa_L = v_L^T M_L v_L/2$, and $\kappa_E = v_E^T M_E v_E/2$.

Thus, if there is no constraint (i.e., $A(q) = 0$) and the control τ can be assigned arbitrarily (i.e., fully actuated), we can achieve the formation-maneuver decoupling by simply canceling out $C_{LE}v_E$ and $C_{EL}v_L$ and drive the formation/maneuver aspects simultaneously and separately by controlling the (decoupled) locked and shape systems individually, for which their Lagrangian-like structure and passivity are instrumental (e.g., passivity-based control).

Unfortunately, a direct application of this standard passive decomposition to the nonholonomic system (1) and (2) seems not so promising, as shown by the presence of $\Delta_\top^T A^T \lambda, \Delta_\perp^T A^T \lambda$ in (11) and (12). In addition to possibly make the control design/analysis complicated, these constraint terms may impose a fundamental restriction on the formation-maneuver decoupling. This is because they may create *noncancelable* energy coupling between the locked and shape systems via the constraint. To better see this, observe the following; from (11) and (12) with Proposition 1, we have

$$\frac{d\kappa_L}{dt} = -v_L^T C_{LE} v_E - v_L^T \Delta_\top^T A^T \lambda + (\tau_L + f_L)^T v_L$$
$$\frac{d\kappa_E}{dt} = -v_E^T C_{EL} v_L - v_E^T \Delta_\perp^T A^T \lambda + (\tau_E + f_E)^T v_E \tag{14}$$

where, from item 3 of Proposition 1, (1), and (10)

$$v_L^T C_{LE} v_E + v_E^T C_{EL} v_L = v_L^T [C_{LE} + C_{EL}^T] v_E = 0$$
$$v_L^T \Delta_\top^T A^T \lambda + v_E^T \Delta_\perp^T A^T \lambda = \lambda^T A \dot{q} = 0. \tag{15}$$

This shows that both the inertia-induced coupling (i.e., $C_{LE}v_E$ and $C_{EL}v_L$) and the constraint-induced coupling (i.e., $\Delta_\top^T A^T \lambda$ and $\Delta_\perp^T A^T \lambda$) define conservative locked-shape energy coupling. Yet, although the former is cancelable [i.e., design $(\tau_L, \tau_E) = (C_{LE}v_E, C_{EL}v_E)$, convert it to τ by (10), and project it on $\mathcal{C}^\top$ by (4)], the latter is not. Here, $v_L^T \Delta_\top^T A^T \lambda$ and $v_E^T \Delta_\perp^T A^T \lambda$ are, in general, not individually zero (i.e., weak decomposability—Section III-B). As long as there is such noncancelable locked-shape energy coupling, there will be no hope for the formation-maneuver decoupling and their separate control.

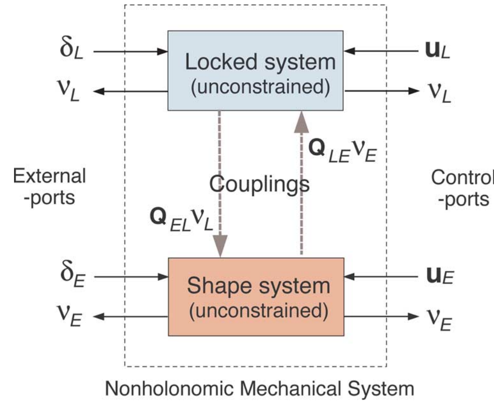

Fig. 2. Energetics of NPD with strong decomposability or with weak decomposability and $\nu_c = 0$.

In the next section, we extend this standard passive decomposition to the nonholonomic mechanical system (1) and (2) so that we can still decouple its formation and maneuver aspects from each other and control them simultaneously and separately to the extent permissible by the nonholonomic constraint (1) and the formation map h.

III. Nonholonomic Passive Decomposition

A. *Nonholonomic Passive Decomposition With Strong Decomposability*

Let us start with the definition of strong decomposability [12], with which the nonholonomic mechanical system (1) and (2) still admits locked-shape decomposition similar to the unconstrained case of standard passive decomposition.

Definition 1: We say that the nonholonomic mechanical system (1) and (2) under the formation map h (7) possesses strong decomposability if

$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp) \quad \forall q \in \mathcal{M}. \tag{16}$$

See item 3 of Lemma 1 for some sufficient conditions for this strong decomposability. This strong decomposability also implies the split of the dual-space $\mathcal{C}^\top$ s.t.

$$\mathcal{C}^\top = (\mathcal{C}^\top \cap \Omega^\top) \oplus (\mathcal{C}^\top \cap \Omega^\perp) \quad \forall q \in \mathcal{M}.$$

Then, we can write $\dot{q}$ and τ s.t.

$$\dot{q} = \underbrace{[\mathcal{D}_\top \cap \Delta_\top \quad \mathcal{D}_\top \cap \Delta_\perp]}_{=:\mathcal{V}(q)} \begin{pmatrix} \nu_L \\ \nu_E \end{pmatrix}$$

$$\tau = \underbrace{[(\mathcal{C}_\top \cap \Omega_\top)^T \quad (\mathcal{C}_\top \cap \Omega_\perp)^T]}_{=:\mathcal{W}^T(q)} \begin{pmatrix} u_L \\ u_E \end{pmatrix} \tag{17}$$

where, similar to (10), each block of $\mathcal{V}(q), \mathcal{W}(q)$ identifies its corresponding vector space. To preserve the mechanical power (i.e., $\tau^T \dot{q} = u_L^T \nu_L + u_E^T \nu_E$), we also enforce $\mathcal{W}(q)\mathcal{V}(q) = I$, which can be achieved by scaling/permutating $\mathcal{W}(q), \mathcal{V}(q)$.

By applying (17), we can then decompose the original Lagrange–D'Alembert dynamics (2) into

$$D_L(q)\dot{\nu}_L + Q_L(q,\dot{q})\nu_L + Q_{LE}(q,\dot{q})\nu_E = u_L + \delta_L \tag{18}$$

$$D_E(q)\dot{\nu}_E + Q_E(q,\dot{q})\nu_E + Q_{EL}(q,\dot{q})\nu_L = u_E + \delta_E \tag{19}$$

where $f =: \mathcal{W}^T(q)[\delta_L; \delta_E]$, $\text{diag}[D_L, D_E] := \mathcal{V}^T M \mathcal{V}$ (from the orthogonality of $\mathcal{D}^\top \cap \Delta^\top$ and $\mathcal{D}^\top \cap \Delta^\perp$ w.r.t. the $M(q)$-metric), and

$$\begin{bmatrix} Q_L & Q_{LE} \\ Q_{EL} & Q_E \end{bmatrix} := \mathcal{V}^T \left[M\dot{\mathcal{V}} + C\mathcal{V} \right]. \tag{20}$$

Here, the dynamics of ν_L in (18) is the *unconstrained* locked system, since it is the locked dynamics of v_L further projected to the unconstrained $\mathcal{D}^\top$. Similarly, the dynamics of ν_E in (19) is the *unconstrained* shape system.

Theorem 1: Consider the nonholonomic mechanical system (1) and (2) with the formation map h (7) and the strong decomposability (16). Then, we can decompose its Lagrange–D'Alembert dynamics (2) into (18) and (19), where we have the following.

1) D_L and D_E are symmetric and positive definite.
2) $\dot{D}_L - 2Q_L$ and $\dot{D}_E - 2Q_E$ are skew symmetric.
3) $Q_{LE} = -Q_{EL}^T$.
4) Kinetic energy and power are decomposed s.t.

$$\kappa(t) = \kappa_L(t) + \kappa_E(t), \quad \tau^T\dot{q} = u_L^T\nu_L + u_E^T\nu_E$$

where $\kappa_L(t) = \nu_L^T D_L \nu_L/2$, and $\kappa_E(t) = \nu_E^T D_E \nu_E/2$.

Proof: Here, we prove only parts of items 2–4, since the rests are either easy to prove or straightforward to deduce. First, observe that, from (20)

$$\begin{aligned} &\begin{bmatrix} \dot{D}_L - 2Q_L & -2Q_{LE} \\ -2Q_{EL} & \dot{D}_E - 2Q_E \end{bmatrix} \\ &= \frac{d[\mathcal{V}^T M \mathcal{V}]}{dt} - 2\mathcal{V}^T[M\dot{\mathcal{V}} + C\mathcal{V}] \\ &= \mathcal{V}^T[\dot{M} - 2C]\mathcal{V} + \dot{\mathcal{V}}^T M \mathcal{V} - \mathcal{V}^T M \dot{\mathcal{V}} \end{aligned}$$

which is skew symmetric. This proves items 2–3. Kinetic energy and power decompositions can also be proved by using $\dot{q}$ in (17) for $\kappa(t) = \dot{q}^T M \dot{q}/2$ and (17) with $\mathcal{W}\mathcal{V} = I$. ■

Therefore, with the strong decomposability (16), similar to the unconstrained case of standard passive decomposition in Section II-C, the nonholonomic mechanical system (1) and (2) can still be decomposed into: 1) (unconstrained) shape system of ν_E, describing the formation aspect; 2) (unconstrained) locked system of ν_L, representing the maneuver aspect; and 3) inertia-induced coupling $Q_{LE}\nu_E, Q_{EL}\nu_L$ between them, which are functions of (usually accessible) $q, \dot{q}$ and energetically conservative (i.e., $\nu_L^T Q_{LE}\nu_E + \nu_E^T Q_{EL}\nu_L = 0$ from item 3 of Theorem 1). Using (18) and (19) with Theorem 1, we can also show that

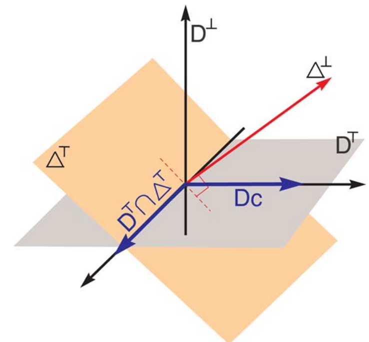

Fig. 3. Example of $\mathcal{D}^\top \neq (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp)$. In this case, $\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus \mathcal{D}^c$ with $\mathcal{D}^\top \cap \Delta^\perp = \emptyset$.

$$\frac{d\kappa_L}{dt} = -\nu_L^T Q_{LE}\nu_E + (u_L + \delta_L)^T \nu_L$$

$$\frac{d\kappa_E}{dt} = -\nu_E^T Q_{EL}\nu_L + (u_E + \delta_E)^T \nu_E \tag{21}$$

revealing the energetic structure of (18) and (19), as shown in Fig. 2, similar to that of standard passive decomposition [3].

Canceling out $Q_{LE}\nu_E, Q_{EL}\nu_L$ and controlling the (decoupled) locked/shape systems in (18) and (19), we can then achieve formation-maneuver decoupling and their separate/simultaneous control, even for the nonholonomic mechanical system (1) and (2). The locked and shape systems' inherited Lagrangian-like structure and passivity in (18) and (19) can also be utilized for this (e.g., passivity-based control). Note that $M(q)$-orthogonality is again crucial here, since, if not, the locked and shape systems in (18) and (19) will be coupled via acceleration channels, which are usually not cancelable in practice.

At this point, the following question may arise: What happened to the noncancelable locked-shape energy coupling via the constraint in (14)? It turns out that such constraint-induced locked-shape coupling disappears under the strong decomposability (16). This can be shown, s.t., by equating (10) and (17) with $A \in \mathcal{C}^\perp$

$$v_L^T \Delta_\top^T A^T = \nu_L^T (\mathcal{D}_\top \cap \Delta_\top)^T A^T = 0$$

$$v_E^T \Delta_\perp^T A^T = \nu_E^T (\mathcal{D}_\top \cap \Delta_\perp)^T A^T = 0 \tag{22}$$

i.e., $v_L^T \Delta_\top^T A^T \lambda = 0$ and $v_E^T \Delta_\perp^T A^T \lambda = 0$ individually in (14), implies no locked-shape energy coupling via the constraint. This, however, does not hold for the more general case of weak decomposability, as discussed in the following.

B. Nonholonomic Passive Decomposition With Weak Decomposability

Although it is very tempting to believe so from (3) and (9), the strong decomposability (16) is not always granted. This is because some of the directions of $\Delta^\top$ or $\Delta^\perp$ can be cut off by the $\cap$-operation (with $\mathcal{D}^\top$); thus, with those directions missing, $\mathcal{D}^\top \cap \Delta^\top$ and $\mathcal{D}^\top \cap \Delta^\perp$ cannot span the whole $\mathcal{D}^\top$-space (see Fig. 3). Let us denote this (missing) quotient space in $\mathcal{D}^\top$ by

$\mathcal{D}^c$, which is orthogonal to $\mathcal{D}^\top \cap \Delta^\top$ and $\mathcal{D}^\top \cap \Delta^\perp$ w.r.t. the $M(q)$-metric. Then, we can write

$$\mathcal{D}^\top = \text{span}\{\alpha_1, \ldots, \alpha_s, \gamma_1, \ldots, \gamma_l, \beta_1, \ldots, \beta_r\} \tag{23}$$

where α_i, β_i, and γ_i are, respectively, the basis of $\mathcal{D}^\top \cap \Delta^\top$, $\mathcal{D}^\top \cap \Delta^\perp$, and $\mathcal{D}^c$ at q. Here, from our construction, $\gamma_i \in \mathcal{D}^\top$, yet

$$\gamma_i \notin \text{span}\{\alpha_1, \ldots, \alpha_s, \beta_1, \ldots, \beta_r\} \tag{24}$$

and, since $\gamma_i \notin \mathcal{D}^\top \cap \Delta^\top$ and $\gamma_i \notin \mathcal{D}^\top \cap \Delta^\perp$, we can write $\gamma_i = \gamma_i^\top + \gamma_i^\perp$, where $\gamma_i^\top \in \Delta^\top$ and $\gamma_i^\perp \in \Delta^\perp$. The following Lemma 1 summarizes some key properties of γ_i.

Lemma 1: Consider $\gamma_i = \gamma_i^\top + \gamma_i^\perp$ in (23) with $\gamma_i^\top \in \Delta^\top$ and $\gamma_i^\perp \in \Delta^\perp$. Then, we have the following.

1) If $\gamma_i \neq 0$, $\gamma_i^\top \neq 0$ and $\gamma_i^\perp \neq 0$.
2) If $\gamma_i \neq 0$, $\gamma_i^\top \notin \mathcal{D}^\top$ and $\gamma_i^\perp \notin \mathcal{D}^\top$.
3) If $\Delta^\top \subset \mathcal{D}^\top$ or $\Delta^\perp \subset \mathcal{D}^\top$, $\gamma_i = 0$.

Proof: For the first item, suppose $\gamma_i \neq 0$, but $\gamma_i^\top = 0$. Then, $\gamma_i = \gamma_i^\perp \in \mathcal{D}^\top \cap \Delta^\perp$, contradictory to (24). Impossibility of $\gamma_i \neq 0$, but $\gamma_i^\perp = 0$ can be shown similarly. For the second item, suppose $\gamma_i^\top \in \mathcal{D}^\top$ (or $\gamma_i^\perp \in \mathcal{D}^\top$, respectively). Then, $\gamma_i^\perp \in \mathcal{D}^\top$ (or $\gamma_i^\top \in \mathcal{D}^\top$, respectively), since $\gamma_i \in \mathcal{D}^\top$. This then implies $\gamma_i^\top \in \mathcal{D}^\top \cap \Delta^\top$ and $\gamma_i^\perp \in \mathcal{D}^\top \cap \Delta^\perp$, contradicting (24). For the last item, suppose $\Delta^\top \subset \mathcal{D}^\top$. Then, we have $\gamma_i^\top \in \Delta^\top \subset \mathcal{D}^\top$, which is possible only with $\gamma_i = 0$ (item 2). The same argument also holds for $\Delta^\perp \subset \mathcal{D}^\top$. ■

We can then see from item 1 of Lemma 1 that any motion in $\mathcal{D}^c$ will simultaneously incur the motion both in $\Delta^\top$ (i.e., maneuver aspect) and $\Delta^\perp$ (i.e., formation aspect). For the maneuver-formation decoupling, thus, we need to regulate any motion in $\mathcal{D}_c$. For the strong decomposability (16), this $\mathcal{D}^c$ is an empty set. Yet, for usual nonholonomic mechanical systems (1) and (2) with a practically useful formation map h (7), $\mathcal{D}^c$ is typically nonempty. To describe this more general case, we now introduce the definition of weak decomposability.

Definition 2: We say that the nonholonomic mechanical system (1) and (2) under the formation map h (7) possesses weak decomposability if

$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp) \oplus \mathcal{D}^c \quad \forall q \in \mathcal{M} \tag{25}$$

where $\mathcal{D}^c \subset \mathcal{D}^\top$ is $M(q)$-orthogonal to $\mathcal{D}^\top \cap \Delta^\top$ and $\mathcal{D}^\top \cap \Delta^\perp$ and contained neither in $\Delta^\top$ nor in $\Delta^\perp$.

Then, similar to (17), we can write

$$\dot{q} = \underbrace{[\,\mathcal{D}_\top \cap \Delta_\top \quad \mathcal{D}_c \quad \mathcal{D}_\top \cap \Delta_\perp\,]}_{=:\mathcal{V}_c(q)} \begin{pmatrix} \nu_L \\ \nu_c \\ \nu_E \end{pmatrix}$$

$$\tau = \underbrace{[\,(\mathcal{C}_\top \cap \Omega_\top)^T \quad C_c^T \quad (\mathcal{C}_\top \cap \Omega_\perp)^T\,]}_{=:\mathcal{W}_c^T(q)} \begin{pmatrix} u_L \\ u_c \\ u_E \end{pmatrix} \tag{26}$$

where each block of $\mathcal{V}_c(q), \mathcal{W}(q)$ identifies its corresponding space. We also set $\mathcal{W}_c(q)$ and $\mathcal{V}_c(q)$ s.t. $\mathcal{W}_c(q)\mathcal{V}_c(q) = I$. Applying (17), we can also decompose the Lagrange–D'Alembert dynamics (2) s.t.

$$\begin{bmatrix} D_L & 0 & 0 \\ 0 & D_c & 0 \\ 0 & 0 & D_E \end{bmatrix} \begin{pmatrix} \dot{\nu}_L \\ \dot{\nu}_c \\ \dot{\nu}_E \end{pmatrix} + \begin{bmatrix} Q_L & Q_{Lc} & Q_{LE} \\ Q_{cL} & Q_c & Q_{cE} \\ Q_{EL} & Q_{Ec} & Q_E \end{bmatrix} \begin{pmatrix} \nu_L \\ \nu_c \\ \nu_E \end{pmatrix} = \begin{pmatrix} u_L \\ u_c \\ u_E \end{pmatrix} + \begin{pmatrix} \delta_L \\ \delta_c \\ \delta_E \end{pmatrix} \tag{27}$$

where, similar to (18) and (19), $\text{diag}[D_L, D_c, D_E] := \mathcal{V}_c^T M \mathcal{V}_c$, $f =: \mathcal{W}_c^T(q)[\delta_L; \delta_c; \delta_E]$, and

$$\begin{bmatrix} Q_L & Q_{Lc} & Q_{LE} \\ Q_{cL} & Q_c & Q_{cE} \\ Q_{EL} & Q_{Ec} & Q_E \end{bmatrix} := \mathcal{V}_c^T[M\dot{\mathcal{V}}_c + C\mathcal{V}_c]. \tag{28}$$

The following Theorem 2 can be proved similar to Theorem 1.

Theorem 2: Consider the nonholonomic mechanical system (1) and (2) with the formation map h (7) and the weak decomposability (25). Then, we can decompose its Lagrange–D'Alembert dynamics (2) into (27), where we have the following.

1) D_L, D_c, and D_E are symmetric and positive definite.
2) $\dot{D}_L - 2Q_L, \dot{D}_E - 2Q_E$, and $\dot{D}_c - 2Q_c$ are skew symmetric.
3) $Q_{\alpha\beta} = -Q_{\beta\alpha}^T$, $\alpha, \beta \in \{L, c, E\}$.
4) Kinetic energy and power are decomposed s.t.

$$\kappa(t) = \frac{1}{2}\nu_L^T D_L \nu_L + \frac{1}{2}\nu_c^T D_c \nu_c + \frac{1}{2}\nu_E^T D_E \nu_E$$
$$\tau^T \dot{q} = u_L^T \nu_L + u_c^T \nu_c + u_E^T \nu_E.$$

Thus, with the weak decomposability (25), the nonholonomic mechanical system (1) and (2) can be decomposed into: 1) shape system of ν_E in $\mathcal{D}^\top \cap \Delta^\perp$, specifying the formation aspect; 2) locked system of ν_L in $\mathcal{D}^\top \cap \Delta^\top$, describing the maneuver aspect; 3) quotient system of ν_c in $\mathcal{D}_c$, whose motion simultaneously perturbing the formation/maneuver aspects; and 4) energetically conservative inertia-induced coupling $Q_{\alpha\beta}\nu_\beta$ among these three systems. Then, canceling out the coupling $Q_{\alpha\beta}\nu_\beta$, regulating the quotient system s.t. $\nu_c = 0$, and controlling the (decoupled) locked and shape systems individually, we can achieve the formation-maneuver decoupling and their separate control, even with the weak decomposability (25). The inherited Lagrangian-like structure and passivity of the locked, shape, and quotient systems can again facilitate their control design/analysis (e.g., passivity-based control of Section IV).

Note that regulating the quotient system (i.e., $\nu_c = 0$) is crucial for the maneuver-formation decoupling, since any nonzero ν_c will perturb both the maneuver and formation aspects simultaneously (item 1 of Lemma 1). From the fact that $\mathcal{D}^c \subset \mathcal{D}^\top$ and the dynamics decomposition (27), we can also see that 1) any quotient control u_c in (27) will be fully effective without being restricted by the nonholonomic constraint (1); and 2) any locked/shape controls u_L, u_E will not perturb the (decoupled) quotient system. Of course, having to dedicate some control directions (i.e., along $\mathcal{C}^c$) solely for the quotient system regulation would likely negatively affect the system's (formation/maneuver) controllability (see Section IV).

The (perturbing) role of the quotient system can also be understood by observing the following energetics. From item 1 of Lemma 1, we can write

$$\mathcal{D}_c \nu_c = \Delta_\top^c \nu_c + \Delta_\perp^c \nu_c \tag{29}$$

where $\mathcal{D}_c$, $\Delta_\top^c$ and $\Delta_\perp^c$ are, respectively, associated with γ_i, $\gamma_i^\top$, and $\gamma_i^\perp$. Then, similar to (15), we have

$$\lambda^T A(q)\mathcal{D}_c\nu_c = \lambda^T A(q)\Delta_\top^c \nu_c + \lambda^T A(q)\Delta_\perp^c\nu_c = 0$$

where, in general, $\lambda^T A(q)\Delta_\top^c\nu_c \neq 0$, and $\lambda^T A(q)\Delta_\perp^c\nu_c \neq 0$, since neither $\Delta_\top^c$ nor $\Delta_\perp^c$ is contained in $\mathcal{D}^\top$ (item 2 of Lemma 1). This shows that: 1) in contrast with the strong decomposability case (22), through $\lambda^T A(q)\Delta_\top^c\nu_c$ and $\lambda^T A(q)\Delta_\perp^c\nu_c$, we have *noncancelable* locked-shape energy coupling via the nonholonomic constraint (15) and the quotient system serves as its conduit; and 2) if we enforce $\nu_c = 0$, the quotient system vanishes, the noncancelable energy coupling through it ceases to flow, and the nonholonomic mechanical system will reduce to the energetic structure of Fig. 2.

Note that the results of Section III—strong/weak decomposabilities (16), (25), and their corresponding NPDs (18), (19), (27)—still hold whether we have full control in $\mathcal{C}^\top$ or not, although such full control in $\mathcal{C}^\top$ can expedite/simplify control design and analysis, as shown in Section IV.

IV. Controllability and Control Design

Our discussion on the controllability and control design in this section is based on the assumption that we have full control in $\mathcal{C}^\top$ for (3), or equivalently, $u \in \Re^{n-p}$ in (4) can be arbitrarily assigned. This means that we rule out the control underactuation in $\mathcal{C}^\top$, which we leave for future research.

As shown in Section III, applying NPD, canceling out inertia-induced coupling, regulating the quotient system, and individually controlling (decoupled) locked and shape systems, we can drive the formation and maneuver aspects simultaneously and separately. Yet, possible maneuver and formation behaviors may still be fundamentally limited by the nonholonomic constraint (1). The question of immediate interest would then be the following: Is it possible to drive the maneuver (or formation, respectively) aspect without being hindered by the nonholonomic constraint (1), while leaving the formation (or maneuver, respectively) aspect intact? For this, we first define the following maneuver decoupled controllability (or d-controllability).

Definition 3: We say that the nonholonomic mechanical system (1) and (2) with the formation map h (7) is maneuver d-controllable if

$$\overline{\text{span}}\{\mathcal{D}^\top \cap \Delta^\top\} = \Delta^\top \quad \forall q \in \mathcal{M} \tag{30}$$

where $\overline{\text{span}}\{\star\}$ denotes the involute closure of $\star$ [8].

This maneuver d-controllability implies that, by moving only along $\mathcal{D}^\top \cap \Delta^\top$, the (unconstrained) locked system can reach any point on the level set $\mathcal{H}_{h(q)}$ (i.e., motion is maximal on $\mathcal{H}_{h(q)}$) without perturbing the formation aspect $h(q)$, even in the presence of the nonholonomic constraint (1). This definition, although seemingly kinematic, is indeed applicable here, since the locked system is kinematically reducible/controllable with full control in $\mathcal{C}^\top \cap \Omega^\top$ [17], [18]. It also generalizes the (kinematic) motion feasibility condition [11] (i.e., $\mathcal{D}^\top \cap \Delta^\top \neq \emptyset$) to the (dynamic) nonholonomic system (2) (as well as to the kinematic system in Section V). The dual notion of (30) is then defined as follows.

Definition 4: We say that the nonholonomic mechanical system (1) and (2) under the formation map h (7) is formation d-controllable if

$$\overline{\text{span}}\left\{\frac{\partial h}{\partial q}(\mathcal{D}^\top \cap \Delta^\perp)\right\} = T_{h(q)}\mathcal{N} \quad \forall q \in \mathcal{M} \tag{31}$$

where $\partial h/\partial q$ acts as push forward from $T_q\mathcal{M}$ to $T_{h(q)}\mathcal{N}$ [15].

This formation d-controllability (31) implies that, by moving the shape system only along $\mathcal{D}^\top \cap \Delta^\perp$, we can achieve *any* formation aspect $h(q)$ on the formation manifold $\mathcal{N}$, without perturbing the maneuver aspect (i.e., locked system), even in the presence of the nonholonomic constraint (1). Because of the same reason as earlier, this seemingly kinematic definition is applicable here for the (dynamic) shape system.

If the system is formation and maneuver d-controllable (30) and (31), we can then not only control its formation and maneuver aspects individually and separately but can also achieve any desired formation/maneuver aspects (on $\mathcal{N}/\mathcal{H}_{h(q)}$) without being restricted by the nonholonomic constraint (1) in the sense of Chow [1]. This also holds whether the system is strong or weak decomposable, although, for the weak decomposable case, loss of control directions in $\mathcal{C}^c$ (for quotient system regulation) would likely impair the system's d-controllabilities.

Assuming both the maneuver and formation aspects be d-controllable, we now design passivity-based control laws to achieve 1) $h(q) \to h_d \in \mathcal{N}$, where h_d represents a certain desired (constant) formation aspect; and 2) $\nu_L(t) \to \nu_L^d(t)$, where $\nu_L^d(t)$ is the (smooth/bounded) velocity profile, encoding a certain desired maneuver behavior on the level set $\mathcal{H}_{h_d}$. The formation d-controllability then implies that we can choose h_d to be any point on the formation manifold $\mathcal{N}$, whereas the maneuver d-controllability that $\nu_L^d(t)$ can be defined to reach any point on $\mathcal{H}_{h_d}$.

First, for the case of strong decomposability, we design the control u_L, u_E in (18) and (19) s.t.

$$\begin{aligned} u_L &= Q_{LE}\nu_E + D_L\dot{\nu}_L^d + Q_L\nu_L^d - B_L(\nu_L - \nu_L^d) - \delta_L \\ u_E &= Q_{EL}\nu_L - B_E\nu_E - S_E\left[\frac{\partial\varphi_E}{\partial h}\right]^T - \delta_E \end{aligned} \tag{32}$$

where B_L and B_E are positive-definite/symmetric gain matrices; $\partial\varphi_E/\partial h \in \Re^{m\times n}$ is the one form of a positive/smooth potential $\varphi_E(h)$ defined on the formation manifold $\mathcal{N}$ s.t. it attains the global minimum at $h = h_d$ and $\partial\varphi_E/\partial h = 0$ if and only if $h = h_d$, and

$$S_E(q) := (\mathcal{D}_\top \cap \Delta_\perp)^T \left[\frac{\partial h}{\partial q}\right]^T \tag{33}$$

which is a "fat" matrix, showing the elimination of some control directions by the nonholonomic constraint (1). Here, we design $S_E[\partial\varphi_E/\partial h]^T$ in (32) s.t.; first define $\tau_E' := -[\partial\varphi_E/\partial h]^T$ on $\mathcal{N}$, while assuming the choice of the paragraph after (10) with

$v_E = dh/dt$ and $\Omega_\perp = \partial h/\partial q$ to have $\tau_E'^T v_E = -d\varphi_E/dt$, and project this τ_E' into u_E' by using (10) and (17).

On the other hand, for the case of weak decomposability, we design the control u_L, u_c, u_E in (27) s.t.

$$\begin{aligned} u_L &= Q_{Lc}\nu_c + Q_{LE}\nu_E \\ &\quad + D_L\dot{\nu}_L^d + Q_L\nu_L^d - B_L(\nu_L - \nu_L^d) - \delta_L \\ u_c &= Q_{cL}\nu_L + Q_{cE}\nu_E - B_c\nu_c - \delta_c \\ u_E &= Q_{EL}\nu_L + Q_{Ec}\nu_c - B_E\nu_E - S_E\left[\frac{\partial\varphi_E}{\partial h}\right]^T - \delta_E \end{aligned} \tag{34}$$

where B_c is a positive-definite/symmetric gain matrix.

For the case of strong decomposability, we then have the usual passivity relation for the potential $\varphi_E(h)$ s.t.

$$\nu_E^T S_E\left[\frac{\partial\varphi_E}{\partial h}\right]^T = \frac{\partial\varphi_E}{\partial h}\frac{\partial h}{\partial q}(\mathcal{D}_\top \cap \Delta_\perp)\nu_E = \frac{d\varphi_E}{dt} \tag{35}$$

where we use $dh/dt = (\partial h/\partial q)\dot{q} = (\partial h/\partial q)(\mathcal{D}_\top \cap \Delta_\perp)\nu_E$ from (17). This passivity relation (35) does not hold for the case of weak decomposability, since we have

$$\nu_E^T S_E\left[\frac{\partial\varphi_E}{\partial h}\right]^T = \frac{d\varphi_E}{dt} - \frac{\partial\varphi_E}{\partial h}\frac{\partial h}{\partial q}\Delta_\perp^c\nu_c$$

from (26) and (29). This implies that, for the potential control of φ_E in (34) to properly work, we need to enforce $\nu_c = 0$, as done in the following theorem.

Theorem 3: Consider the nonholonomic mechanical system (1) and (2) with the formation map h and the control (32) for strong decomposability or (34) for weak decomposability. Suppose that $\dot{q}(0) = 0$; and $\mathcal{V}(q)$ in (17), or $\mathcal{V}_c(q)$ in (26), and their partial derivatives are bounded $\forall q$. Then, $\dot{q} \in \mathcal{L}_\infty$, $\nu_L(t) \to \nu_L^d(t)$, $\nu_c(t) = 0$, and $\varphi_E(t) \le \varphi_E(0)\ \forall t \ge 0$.

Suppose further that (a) $D_E(q), Q_E(q,\dot{q}), Q_L(q,\dot{q})$, and their partial derivatives are bounded if $\dot{q}$ is bounded; (b) $\partial h/\partial q$, $\partial\varphi_E/\partial h$, $S_E(q)$, and their partial derivatives are bounded if $\varphi_E(h)$ is bounded; and (c) $\nu_L^d(t)$ drives $q(t)$ in such a way that, for all $t' \ge 0$, there exists a bounded duration $\Delta t > 0$, s.t.

$$S_E(q(t))\left[\frac{\partial\varphi_E}{\partial h}\right]^T = 0 \quad \forall t \in [t', t'+\Delta t] \quad iff \quad \frac{\partial\varphi_E}{\partial h} = 0. \tag{36}$$

Then, we have $h(q) \to h_d$.

Proof: With the skew symmetricity of $\dot{D}_L - 2Q_L$, it is easy to show that, both for the cases of strong and weak decomposability, $\nu_L \to \nu_L^d$ exponentially. In addition, for the weak decomposability case, with u_c in (34), $\nu_c(0) = 0$ and skew-symmetric $\dot{D}_c - 2\dot{Q}_c$, we can show that $\nu_c(t) = 0\ \forall t \ge 0$. With $\nu_c = 0$, (35) then holds even for the weak decomposability. Thus, both for the cases of strong and weak decomposability, using the closed-loop shape dynamics

$$D_E\dot{\nu}_E + Q_E\nu_E + B_E\nu_E + S_E\left[\frac{\partial\varphi_E}{\partial h}\right]^T = 0 \tag{37}$$

with skew-symmetric $\dot{D}_E - 2Q_E$ and (35), we can obtain

$$\frac{d}{dt}(\kappa_E + \varphi_E) = -\nu_E^T B_E \nu_E$$

where $\kappa_E := \nu_E^T D_E \nu_E/2$. Integrating this, we have $\forall t' \ge 0$

$$\kappa_E(t') + \varphi_E(t') - \kappa_E(0) - \varphi_E(0) = -\int_0^{t'} \nu_E^T B_E \nu_E\, dt$$

which implies that 1) $\nu_E \in \mathcal{L}_\infty$, and thus, $\dot{q} \in \mathcal{L}_\infty$ (with $\nu_c = 0$, $\nu_L - \nu_L^d \in \mathcal{L}_\infty$, and bounded $\mathcal{V}, \mathcal{V}_c$); and 2) $\varphi_E(t') \le \varphi_E(0)$ $\forall t' \ge 0$ (with $\kappa_E(t') \ge 0$, and $\kappa_E(0) = 0$).

This integration also shows that $\nu_E \in \mathcal{L}_2 \cap \mathcal{L}_\infty$. Using the assumptions (a) and (b) with bounded $\dot{q}$, ν_E, and φ_E, we can also see from (37) that $\dot{\nu}_E \in \mathcal{L}_\infty$. Then, from Barbalat's lemma [14], $\nu_E \to 0$. Differentiating (37) and using the assumptions (a) and (b) with $\dot{q} \in \mathcal{L}_\infty$ and $\ddot{q} \in \mathcal{L}_\infty$ (from $\dot{\nu}_E \in \mathcal{L}_\infty$; $\nu_c = 0$; $\dot{\nu}_L \in \mathcal{L}_\infty$ from bounded ν_L^d, $\dot{\nu}_L^d$, $\nu_L - \nu_L^d$ and (a); and bounded, e.g., $\mathcal{V}_c, \partial\mathcal{V}_c/\partial q$), we can further show that $\ddot{\nu}_E \in \mathcal{L}_\infty$. This, then, again from Barbalat's lemma, implies $\dot{\nu}_E \to 0$. With $\nu_E \to 0$ and $\dot{\nu}_E \to 0$ in (37), we finally have $S_E[\partial\varphi_E/\partial h]^T \to 0$, which implies $\partial\varphi_E/\partial h \to 0$ from (36), and $h(q) \to h_d$ from the construction of φ_E. ■

The result $\varphi_E(t) \le \varphi_E(0)$ in Theorem 3 implies that, if we start with the desired formation shape $h(0) = h_d$, the control (34) [or (32)] will enforce it thenceforth with $h(t) = h_d\ \forall t \ge 0$. The control (34) [or (32)] also needs to be decoded into u (or τ) in (4) before being applied to the real system (2). For this, we can use the following control decoding map by equating τ in (4) and (26) and multiplying them by $\mathcal{V}_c^T(q)$

$$u = \left(\left[\mathcal{D}_\top \cap \Delta_\top \quad \mathcal{D}_c \quad \mathcal{D}_\top \cap \Delta_\perp\right]^T \mathcal{C}_\top^T\right)^{-1} \begin{pmatrix} u_L \\ u_c \\ u_E \end{pmatrix} \tag{38}$$

where $\mathcal{D}_c$ and u_c vanish for the case of strong decomposability. In (38), the velocity subspaces $\mathcal{D}_\top \cap \Delta_\top$, $\mathcal{D}_c$, and $\mathcal{D}_\top \cap \Delta_\perp$ would already have been computed to achieve NPD [e.g., (27)], and $\mathcal{C}_\top(q)$ is also often easy to identify [e.g., if $\mathcal{D}_\top^T\mathcal{D}_\top = I$, $C_\top := \mathcal{D}_\top^T$ (see Section VI)]. We may obtain a similar relation by multiplying $\mathcal{D}_\top$ to (4) and (26), which, yet, requires computation of $\mathcal{C}_\top \cap \Omega_\top$, $\mathcal{C}_c$, and $\mathcal{C}_\top \cap \Omega_\perp$, that are often unnecessary other than this control decoding purpose.

Note that, although relatively simple, our passivity-based control (34) [or (32)] can attain rather interesting/useful behavior of Theorem 3, due to its utilization of Lagrangian-like structure and passivity of the locked/shape/quotient systems, that our NPD strives to preserve. Other types of control laws/strategies, of course, may also be applied here (e.g., optimal control on $\mathcal{N}$, robust tracking control, and underactuated control in $\mathcal{C}^\top$ [19]). It is also interesting that the condition (36) serves similar to the persistency of excitation condition for adaptive control parameter convergence [20].

V. Extension to Kinematic Nonholonomic Systems

Let us consider the kinematic driftless control system under the nonholonomic constraint (1) as given by

$$\dot{q} = \mathcal{D}_\top u \tag{39}$$

where $q \in \Re^n$ is the configuration, $\mathcal{D}_\top \in \Re^{n\times(n-p)}$ identifies $\mathcal{D}^\top$ of the constraint (1), and $u \in \Re^{n-p}$ is the control input.

Note that this kinematic system (39) assumes velocity u as its control input with no forcing terms showing up. Consequently, we will only have velocity decomposition here and not that of the force, as in Section III.

With the formation map h in (7), similar to (9), we can then define $\Delta^\top$ as the kernel of $\partial h/\partial q$ and $\Delta^\perp$ as its orthogonal complement. We can also define, similar to (25) [or (16)], $\mathcal{D}^\top \cap \Delta^\top$, $\mathcal{D}^\top \cap \Delta^\perp$, and $\mathcal{D}^c$ using set operations among $\Delta^\top$, $\Delta^\perp$, and $\mathcal{D}^\top$. To define orthogonality among these velocity subspaces, we use Euclidean metric here (e.g., $\Delta_\top^T \Delta_\perp = 0$ with $\Delta_\perp = \partial h/\partial q$, $\mathcal{D}_c^T(\mathcal{D}_\top \cap \Delta_\top) = 0$), although other metric is also possible. Recall that, in Section III, we need to use the $M(q)$-metric to define such orthogonality so that, by block-diagonalizing inertia matrices [e.g., diag$[D_L, D_c, D_E]$ in (27)], we can avoid noncancelable acceleration coupling among the locked/shape/quotient systems.

Then, we can write (39) similar to (26) s.t.

$$\dot{q} = \underbrace{[\,\mathcal{D}_\top \cap \Delta_\top \quad \mathcal{D}_c \quad \mathcal{D}_\top \cap \Delta_\perp\,]}_{=:\mathcal{V}_k(q)} \begin{pmatrix} u_L \\ u_c \\ u_E \end{pmatrix} \tag{40}$$

where u_L, u_c, and u_E are the (transformed) velocity control inputs, respectively, for the (unconstrained/kinematic) locked, quotient, and shape systems. Similar to (38), we can also obtain the control decoding map

$$u = (\mathcal{D}_\top^T \mathcal{D}_\top)^{-1} \mathcal{D}_\top^T \mathcal{V}_k \begin{pmatrix} u_L \\ u_c \\ u_E \end{pmatrix}$$

by equating (39) and (40).

The kinematic decomposition (40) and its velocity subspaces carry the same geometric meaning as their counterparts in Sections III and IV, i.e., 1) the kinematic system (39) may be strong decomposable (16) or weak decomposable (25); 2) u_L (or u_E, respectively) represents (unconstrained) maneuver (or formation, respectively) aspect in $\mathcal{D}^\top \cap \Delta^\top$ (or $\mathcal{D}^\top \cap \Delta^\perp$, respectively); 3) any quotient motion u_c in $\mathcal{D}^c$ perturbs both the formation and maneuver aspects with Lemma 1 still hold; and 4) the notions of maneuver/formation d-controllability (30) and (31) are readily applicable for the kinematic system (39). Yet, any aspects related to the inertia, force, and energy (e.g., inertia/constraint-induced coupling and energetics decomposition) cannot be revealed with these kinematic modeling (39) and decomposition (40).

Let us assume that the kinematic system (39) is formation and maneuver d-controllable. We can then design a control law similar to Section IV s.t.

$$u_L = \nu_L^d, u_c = 0, u_E = -S_E(q)\left[\frac{\partial \varphi_E}{\partial h}\right]^T \tag{41}$$

where $\nu_L^d(t)$ is the desired maneuver velocity profile, $\varphi_E(h)$ is the potential similar to that in (32), and $S_E(q)$ is as defined in (33). Then, it is obvious from (40) and (41) that: 1) desired maneuver is achieved with $u_L(t) = \nu_L^d(t)\ \forall t \geq 0$; and 2) quotient system vanishes with $u_c(t) = 0\ \forall t \geq 0$. The following Theorem 4 shows that the above u_E also achieves desired formation aspect $h(q) \to h_d$ for the kinematic system (39) in a manner similar to that of Theorem 3.

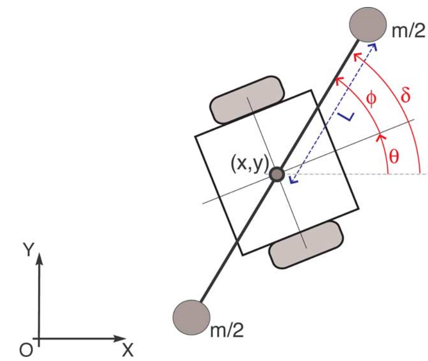

Fig. 4. WMR with 1-DOF arm and two counter-balancing masses.

Theorem 4: Consider the kinematic nonholonomic system (39) with the control (u_L, u_c, u_E) in (41). Suppose that $\partial h/\partial q$, $\partial \varphi_E/\partial h$, $S_E(q)$ satisfy the assumption (b) in Theorem 3, and $\mathcal{V}_k(q)$ in (40) is bounded $\forall q$. Then, $\dot{q} \in \mathcal{L}_\infty$ and $\varphi_E(t) \leq \varphi_E(0)$ $\forall t \geq 0$. Moreover, if $\nu_L^d(t)$ excites the system to satisfy (36), $h(q) \to h_d$.

Proof: With $u_c = 0$, from (40) with (33), we have

$$\frac{d\varphi_E}{dt} = \frac{\partial \varphi_E}{\partial h}\frac{\partial h}{\partial q}\dot{q} = \frac{\partial \varphi_E}{\partial h}\frac{\partial h}{\partial q}(\mathcal{D}_\top \cap \Delta_\perp) u_E = -u_E^T u_E$$

implying $\varphi_E(t) \leq \varphi_E(0)\ \forall t \geq 0$. We then have $u_E \in \mathcal{L}_\infty$ from (41) with the assumption (b) of Theorem 3, and $\dot{q} \in \mathcal{L}_\infty$, with $u_E, u_c(=0), u_L(=\nu_L^d)$, and $\mathcal{V}_k(q)$ in (40) all bounded. Integration of the above equality also shows $u_E \in \mathcal{L}_2 \cap \mathcal{L}_\infty$. Moreover, differentiating u_E in (41) with the assumption (b) of Theorem 3 and bounded $\dot{q}$, we have $\dot{u}_E \in \mathcal{L}_\infty$. This then implies, from Barbalat's lemma [14], that $u_E \to 0$; thus, it follows that $\partial \varphi_E/\partial h \to 0$ from (36) and $h(q) \to h_d$ from the construction of φ_E. ■

Note that the kinematic results in Section V still capture much of the essence of the dynamic results in Sections III and IV, although it is much simpler than them. The maneuver d-controllability (30) also generalizes/completes the motion feasibility condition of [11] (i.e., $\mathcal{D}^\top \cap \Delta^\top \neq \emptyset$). The kinematic results in Section V, however, cannot address dynamics-related phenomena (e.g., effect of inertia/force, power/energetics decomposition), whose effects are often crucial in some important applications (e.g., force controlled assembly, high-speed operation, etc); nor allow us to reveal how to exploit open-loop dynamics of nonholonomic mechanical systems (2) (e.g., parsimonious cancellation of inertia-induced coupling in Section IV), since the kinematic equation (39) itself requires such open-loop dynamics to be perfectly canceled out (e.g., computed torque control) or neglects it (e.g., high-gain control).

Since the main focus of this paper is on the nonholonomic mechanical systems (2) and the results for the kinematic systems (39) in Section V are simpler than those in Sections III and IV, in Section V, we will present numerical examples and illustrate theory only for the former case.

VI. Illustrative Examples

A. *Wheeled Mobile Robot With 1-DOF Arm*

This example is mainly pedagogical to explain the theory. We consider a wheeled mobile robot (WMR) with a 1-DOF arm on it, which has two counter-balancing masses $m/2$ at the distance l and rotates at the WMR's geometric center (x, y) (see Fig. 4). For simplicity, we assume the WMR's geometric center and its center of mass are the same. With the WMR's mass being m_c and its rotational inertia I, we can then write the kinetic energy of this system by

$$\kappa_v = \frac{1}{2}\dot{q}^T \underbrace{\begin{bmatrix} m_c + m & 0 & 0 & 0 \\ 0 & m_c + m & 0 & 0 \\ 0 & 0 & I + ml^2 & ml^2 \\ 0 & 0 & ml^2 & ml^2 \end{bmatrix}}_{=:M} \dot{q}$$

where $q = [x, y, \theta, \phi]^T \in \Re^4$ is the system configuration written w.r.t. the inertial frame (O, X, Y). With the constant M, we can also show that Coriolis matrix $C(q, \dot{q}) = 0$.

The system is under the nonholonomic no-slip constraint

$$\underbrace{[\mathrm{s}\,\theta \quad -\mathrm{c}\,\theta \quad 0 \quad 0]}_{=A(q)}\dot{q} = 0$$

where $\mathrm{s}\,\theta := \sin\theta$, and $\mathrm{c}\,\theta := \cos\theta$. Then, we can find

$$\mathcal{D}^\top = \mathrm{span}\begin{bmatrix} \mathrm{c}\,\theta & \mathrm{s}\,\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{42}$$

where span A denotes the span space of the column vectors of $A \in \Re^{n \times m}$.

Let us consider the following formation map:

$$h(q) = y + l\mathrm{s}\,(\theta + \phi)$$

i.e., y position of one of the rotating masses, with

$$\frac{\partial h}{\partial q} = \Omega^\perp = [0 \quad 1 \quad l\,\mathrm{c}\,\delta \quad l\,\mathrm{c}\,\delta]$$

where $\delta := \theta + \phi$. Then, we can compute

$$\Delta^\top = \mathrm{span}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2l\,\mathrm{c}\,\delta & -1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}^T$$

and

$$\mathcal{D}^\top \cap \Delta^\top = \mathrm{span}\begin{bmatrix} 2l\,\mathrm{c}\,\theta\,\mathrm{c}\,\delta & 2l\,\mathrm{s}\,\theta\,\mathrm{c}\,\delta & -\mathrm{s}\,\theta & -\mathrm{s}\,\theta \\ 0 & 0 & 1 & -1 \end{bmatrix}^T.$$

We can also compute the remaining $\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)$ by solving for $\alpha, \beta \in \Re$ in the following equality:

$$\begin{pmatrix} \mathrm{c}\,\theta \\ \mathrm{s}\,\theta \\ \alpha \\ \beta \end{pmatrix}^T \begin{bmatrix} 2m_t l\,\mathrm{c}\,\theta\,\mathrm{c}\,\delta & 0 \\ 2m_t l\,\mathrm{s}\,\theta\,\mathrm{c}\,\delta & 0 \\ -(I + 2ml^2)\,\mathrm{s}\,\theta & I \\ -2ml^2\,\mathrm{s}\,\theta & 0 \end{bmatrix} = 0$$

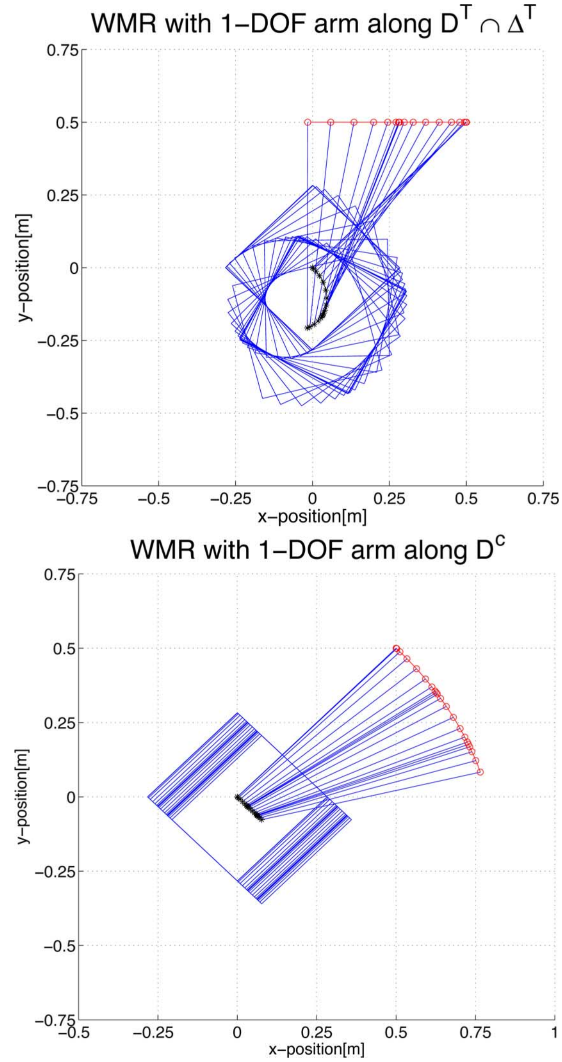

Fig. 5. Motion of WMR with a 1-DOF arm (only one countermass is shown here): along $\mathcal{D}^\top \cap \Delta^\top$ with no change in h (top figure) and along $\mathcal{D}^c$ invoking change both in h and $\Delta^\top$ (e.g., WMR x-motion).

where the left vector represents any vector in $\mathcal{D}^\top$ with arbitrary α, β, while the right matrix is $M(\mathcal{D}^\top \cap \Delta^\top)$ with $m_t = m + m_c$. Here, M is multiplied, since $\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)$ and $\mathcal{D}^\top \cap \Delta^\top$ should be orthogonal with each other w.r.t. the M-metric. Solving this, we then have

$$\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top) = \mathrm{span}\begin{bmatrix} \frac{\mathrm{s}\,2\theta}{2} & \frac{1 - \mathrm{c}\,2\theta}{2} & 0 & \frac{m_t\,\mathrm{c}\,\delta}{ml} \end{bmatrix}^T \tag{43}$$

which may be contained in $\Delta^\perp$ [i.e., strong decomposability (16)] or not [i.e., weak decomposability (25)].

If (43) is contained in $\Delta^\perp$, it should be orthogonal to $\Delta^\top$ w.r.t. the M-metric. However, this is not the case here, since

$$\begin{pmatrix} \frac{\mathrm{s}\,2\theta}{2} \\ \frac{1-\mathrm{c}\,2\theta}{2} \\ 0 \\ \frac{m_t\,\mathrm{c}\,\delta}{ml} \end{pmatrix}^T \begin{bmatrix} m_t & 0 & 0 \\ 0 & 2m_t l\,\mathrm{c}\,\delta & 0 \\ 0 & -I - 2ml^2 & I \\ 0 & -2ml^2 & 0 \end{bmatrix} = \begin{pmatrix} m_t\,\mathrm{c}\,\theta\,\mathrm{s}\,\theta \\ -2m_t l\,\mathrm{c}\,\delta\,\mathrm{c}^2\theta \\ 0 \end{pmatrix}^T$$

is, in general, not zero, where the second matrix is $M\Delta^\top$. This implies that (43) constitutes $\mathcal{D}^c$, $\mathcal{D}^\top \cap \Delta^\perp$ is empty, and the system satisfies the following weak decomposability:

$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus \mathcal{D}^c$$

with $\mathcal{D}_c$ given by (43). This also means that we can drive the system along $\mathcal{D}^\top \cap \Delta^\top$ without perturbing h (i.e., y position of the counter mass), yet we cannot change h (along $\Delta^\perp$) without invoking motion in $\Delta^\top$.

To show this, we perform simulation with $\dot{q}(0) = 0$ and $f(t) = 0 \; \forall t \geq 0$ for (2). We use the following control:

$$u_L = Q_{Lc}\nu_c + \omega_L, \quad u_c = Q_{cL}\nu_L + \omega_c$$

where $\omega_L \in \Re^2$ and $\omega_c \in \Re$ are additional controls (specified as following). Although this control (u_L, u_c) is slightly different from (34), we can see from (27) that it can still decouple the ν_L and ν_c dynamics from each other; also, if $\nu_L(0) = 0$ with $\omega_L = 0$ (or $\nu_c(0) = 0$ with $\omega_c = 0$, respectively), it will enforce $\nu_L(t) = 0$ (or $\nu_c(t) = 0$, respectively) for all $t \geq 0$. This control is then decoded into u via (38), for which we can use $\mathcal{C}_\top := \mathcal{D}_\top^T$, with $\mathcal{D}_\top$ defined as the matrix in (42). This is possible, since $\mathcal{D}_\top^T \mathcal{D}_\top = I = \mathcal{C}_\top \mathcal{D}_\top$ (i.e., $\mathcal{D}_\top^T$ identifies $\mathcal{C}^\top$). See Fig. 5 for simulation results, where: 1) in the top plot, by driving the system only along $\mathcal{D}^\top \cap \Delta^\top$ with a sinusoid ω_L and $\omega_c = 0$, we can move the system without perturbing $h(q)$ (with $\nu_c(t) = 0$ $\forall t \geq 0$); and 2) in the bottom plot, by driving the system along $\mathcal{D}^c$ via a sinusoid ω_c with $\omega_L = 0$, we can change h, yet, at the same time, invoke motion in $\Delta^\top$ (e.g., WMR's motion along the x-axis).

Here, with certain coordinate choices, some of the computed subspaces of NPD may possess singularity [e.g., $\mathcal{D}_c = [\mathrm{c}\,\theta \quad \mathrm{s}\,\theta \quad 0 \quad m_t \mathrm{c}\,\delta / ml\,\mathrm{s}\,\theta]^T$ instead of (43)]. This problem may be solved simply by trying other coordinates or, perhaps, more fundamentally, using coordinate-invariant/differential-geometric derivations of the NPD that we will report in a future publication. See [2] and [3] for such coordinate-free derivations of the standard passive decomposition.

B. Formation/Maneuver Control of Three Wheeled Mobile Robots

Let us consider a team of three 3-DOF dynamic WMRs with its ith robot's dynamics ($i = 1, 2, 3$) given by 1) the nonholonomic constraint (i.e., no-slip condition of wheels)

$$[\mathrm{s}\,\theta_i \quad -\mathrm{c}\,\theta_i \quad 0]\,\dot{q}_i = A_i(\theta_i)\dot{q}_i = 0$$

and 2) the Lagrange–D'Alembert equation

$$M_i(\theta_i)\ddot{q}_i + C_i(\theta_i, \dot{\theta}_i) + \lambda_i A_i^T(\theta_i) = \tau_i + f_i$$

where $q_i := (x_i, y_i, \theta_i) \in \Re^3$ is the configuration composed of the WMR's geometric center position (x_i, y_i) and orientation θ_i w.r.t. the common inertial frame (O, X, Y)

$$M_i(\theta_i) = \begin{bmatrix} m_i & 0 & -m_i d_i \,\mathrm{s}\,\theta_i \\ 0 & m_i & m_i d_i \,\mathrm{c}\,\theta_i \\ -m_i d_i \,\mathrm{s}\,\theta_i & m_i d_i \,\mathrm{c}\,\theta_i & I_i \end{bmatrix}$$

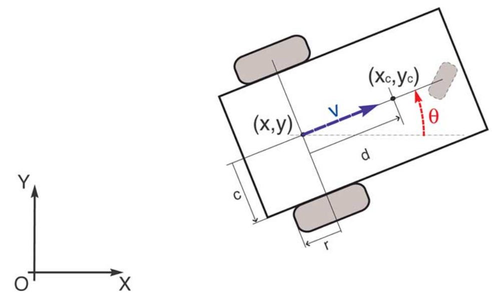

Fig. 6. WMR with the geometric center at (x, y) and the center-of-mass at (x_c, y_c), both w.r.t. the inertial frame (O, X, Y).

$$C_i(\theta_i, \dot{\theta}_i) = \begin{bmatrix} 0 & 0 & -m_i d_i \dot{\theta}_i \,\mathrm{c}\,\theta_i \\ 0 & 0 & -m_i d_i \dot{\theta}_i \,\mathrm{s}\,\theta_i \\ 0 & 0 & 0 \end{bmatrix}$$

with m_i, $I_i = I_o^i + m_i d_i^2$, d being, respectively, the robot's mass, moment of inertia (w.r.t. geometric center), and distance between its geometric and mass centers; and $\tau_i, f_i \in \Re^3$ are the control and external forces (see Fig. 6). Although we consider only three WMRs for simplicity, our derivations/results here are easily extendable for multiple WMRs [21].

By combining each robot's dynamics, we can then write their (product) team dynamics in the form of (1) and (2) with $q := [q_1; q_2; q_3] \in \Re^9$, $M := \mathrm{diag}[M_1, M_2, M_3]$, $C := \mathrm{diag}[C_1, C_2, C_3] \in \Re^{9\times 9}$, $A := \mathrm{diag}[A_1, A_2, A_3] \in \Re^{N\times 3N}$, and $\lambda = [\lambda_1; \lambda_2; \lambda_3] \in \Re^3$. We can also compute the team's unconstrained distribution

$$\mathcal{D}^\top = \mathrm{span}\begin{bmatrix} \mathrm{c}\,\theta_1 & \mathrm{s}\,\theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{c}\,\theta_2 & \mathrm{s}\,\theta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{c}\,\theta_3 & \mathrm{s}\,\theta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

with $\dim \mathcal{D}^\perp = 3$.

Let us consider the following simple yet versatile linear formation map:

$$h := [x_1 - x_2; y_1 - y_2; \theta_1 - \theta_2; x_2 - x_3; y_2 - y_3; \theta_2 - \theta_3]$$

with $h_d = [p_{12}^d; 0; p_{23}^d, 0] \in \Re^6$, i.e., we want the WMRs to make a certain (constant) Cartesian formation shape specified by $p_{12}^d, p_{23}^d \in \Re^2$, with their heading angle θ_i being aligned with each other. Then, we have

$$\Omega^\perp = \mathrm{span}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and also can compute its kernel space $\Delta^\top$ to be

$$\Delta^\top = \text{span}\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T$$

with $\dim \Delta^\perp = 6$.

Now, to make the computation simpler, let us assume that $\theta_1(t) = \theta_2(t) = \theta_3(t) =: \theta(t)\ \forall t \geq 0$. This will be guaranteed by the control (34) with the initial condition $\theta_1(0) = \theta_2(0) = \theta_3(0)$ (see the Corollary 1 below).[2] Then, from the earlier $\mathcal{D}^\top$ and $\Delta^\top$ with $\theta_i = \theta$, we have

$$\mathcal{D}^\top \cap \Delta^\top = \text{span}\begin{bmatrix} \text{c}\,\theta & \text{s}\,\theta & 0 & \text{c}\,\theta & \text{s}\,\theta & 0 & \text{c}\,\theta & \text{s}\,\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T$$

where the first column represents the coordinated (x, y)-motion of the three WMRs, while the second represents their coordinated θ-rotation. We can also compute, similar to Section VI-A, the remaining space $\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)$ by using $[\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)]^T M(\mathcal{D}^\top \cap \Delta^\top) = 0$, where, in this case

$$M(\mathcal{D}^\top \cap \Delta^\top) = \begin{bmatrix} M_1' \\ M_2' \\ M_3' \end{bmatrix} \in \Re^{9\times 2}$$

with

$$M_i' := \begin{bmatrix} m_i\,\text{c}\,\theta & -m_i d_i\,\text{s}\,\theta \\ m_i\,\text{s}\,\theta & m_i d_i\,\text{c}\,\theta \\ 0 & I_i \end{bmatrix} \in \Re^{3\times 2}.$$

Further using the fact that $\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)$ is contained in $\mathcal{D}^\top$ with $\dim[\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top)] = 4$, we can obtain

$$\mathcal{D}^\top - (\mathcal{D}^\top \cap \Delta^\top) = \text{span}\begin{bmatrix} \frac{\text{c}\,\theta}{m_1} & 0 & 0 & 0 \\ \frac{\text{s}\,\theta}{m_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_1} & 0 \\ -\frac{\text{c}\,\theta}{m_2} & \frac{\text{c}\,\theta}{m_2} & 0 & 0 \\ -\frac{\text{s}\,\theta}{m_2} & \frac{\text{s}\,\theta}{m_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{I_2} & \frac{1}{I_2} \\ 0 & -\frac{\text{c}\,\theta}{m_3} & 0 & 0 \\ 0 & -\frac{\text{s}\,\theta}{m_3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{I_3} \end{bmatrix}$$

where the first two columns are wholly contained in $\Delta^\perp$ as they annihilate $M\Delta^\top = \text{diag}[M_1, M_2, M_3]$, while the last two columns are not, implying that they have components both in $\Delta^\top$ and $\Delta^\perp$. Moreover, the first two and the last two columns are orthogonal with each other w.r.t. the M-metric. This implies that the first two columns constitute $\mathcal{D}^\top \cap \Delta^\perp$, the last two $\mathcal{D}^c$ and, consequently, the system possesses weak decomposability (25).

Then, we can construct the NPD matrix $\mathcal{V}_c$ in (26) with the assumption that $\theta_1 = \theta_2 = \theta_3$. We can also see that the system is maneuver d-controllable (30) if $\theta_1 = \theta_2 = \theta_3$, since $\overline{\text{span}}\{\mathcal{D}^\top \cap \Delta^\top\} = \Delta^\top$. This may also be understood as follows. On any level set $\mathcal{H}_{h_d}$, the three WMRs' velocities $\dot{q}_i, \dot{\theta}_i$ are the same as encoded by $\mathcal{D}^\top \cap \Delta^\top$. This means $\mathcal{H}_{h_d}$ can be parameterized by $q_i \in \Re^3$ of any single WMR, which represents the "bulk" motion of the three WMRs glued together (e.g., summing up individual dynamics with $\theta_i = \theta_j$ and $\dot{q}_i = \dot{q}_j$). The motion of this $\dot{q}_i$, however, can reach any point in $\Re^3$, hence, that in $\mathcal{H}_{h_d}$, since $\overline{\text{span}}\{[\text{c}\,\theta \quad \text{s}\,\theta \quad 0]^T, [0 \quad 0 \quad 1]^T\} = \Re^3$, which is equivalent to $\overline{\text{span}}\{\mathcal{D}^\top \cap \Delta^\top\} = \Delta^\top$.

On the other hand, we can also show that, with $\theta_1 = \theta_2 = \theta_3$, the system is formation d-controllable (31) w.r.t. $p_{12} := [x_1 - x_2; y_1 - y_2] \in \Re^2$ and $p_{23} := [x_2 - x_3; y_2 - y_3] \in \Re^2$, i.e., we can achieve *any* Cartesian formation shape (p_{12}, p_{23}) by recruiting the motion of $\theta := \theta_i$ without perturbing the maneuver aspect. To see this, observe that

$$\frac{\partial h}{\partial q}\left[\mathcal{D}^\top \cap \Delta^\perp\right] = \begin{bmatrix} \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\text{c}\,\theta & -\frac{1}{m_2}\text{c}\,\theta \\ \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\text{s}\,\theta & -\frac{1}{m_2}\text{s}\,\theta \\ 0 & 0 \\ -\frac{1}{m_2}\text{c}\,\theta & \left(\frac{1}{m_2} + \frac{1}{m_3}\right)\text{c}\,\theta \\ -\frac{1}{m_2}\text{s}\,\theta & \left(\frac{1}{m_2} + \frac{1}{m_3}\right)\text{s}\,\theta \\ 0 & 0 \end{bmatrix}$$

where the third/sixth rows are zero from $\theta_1 = \theta_2 = \theta_3$, while the first–second and fourth–fifth rows, respectively, represent motions for p_{12} and p_{23}. We can then show that, combined with the θ-motion, these p_{12}, p_{23} motions can achieve any Cartesian formation shape $(p_{12}, p_{23}) \in \Re^4$, since we have

$$\overline{\text{span}}\begin{bmatrix} \lambda_1\,\text{c}\,\theta & \lambda_1\,\text{s}\,\theta & 0 & 0 & 0 \\ 0 & 0 & \lambda_2\,\text{c}\,\theta & \lambda_2\,\text{s}\,\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T = \Re^5$$

for any $\lambda_1, \lambda_2 \neq 0$, where the first two columns are obtained from the aforementioned $(\partial h/\partial q)(\mathcal{D}^\top \cap \Delta^\perp)$ to, respectively, specify the motions for p_{12} and p_{23}, while the last column specifies the θ-motion.

With these formation and maneuver d-controllabilities, we apply the control (u_L, u_c, u_E) in (34) to the three WMRs with 1) $\varphi_E(h) := (h - h_d)^T K_E (h - h_d)/2$, where $K_E \in \Re^{6\times 6}$ is a positive-definite/symmetric matrix; and 2) $\nu_L^d(t) := [v_L^d(t), \dot{\theta}_d(t)]$, where $\dot{\theta}_d(t)$ is designed s.t. $\theta(t)$ oscillates between two different values to enforce the condition (36) [see (44)]. This (u_L, u_c, u_E) is then decoded using (38) into the control u_i for each WMR. For this, similar to Section VI-A, we can

[2] This can also be achieved by extracting *unconstrained* θ_i-dynamics from each robot [22] and applying standard passive decomposition to them [4] while using NPD to the remaining *constrained* (x_i, y_i)-dynamics.

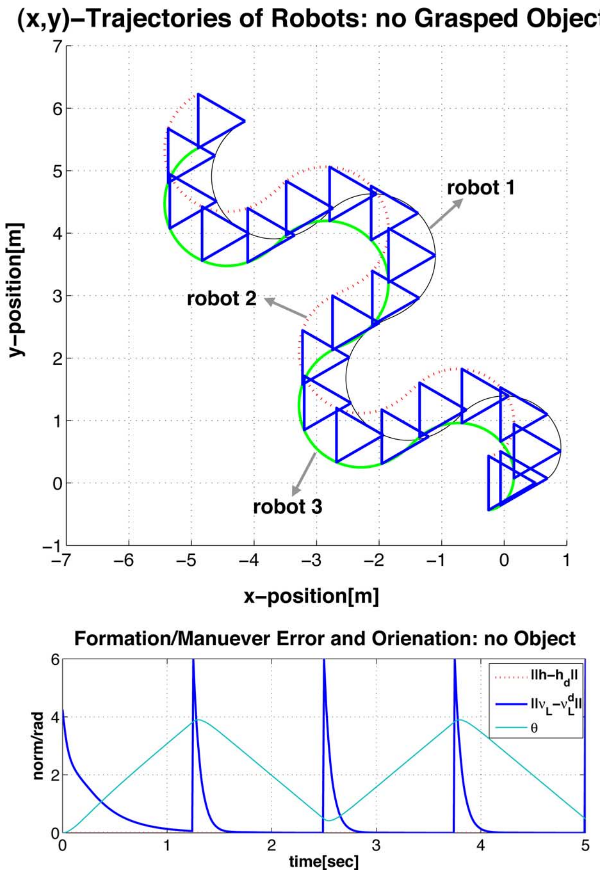

Fig. 7. Driving with formation keeping of three WMRs.

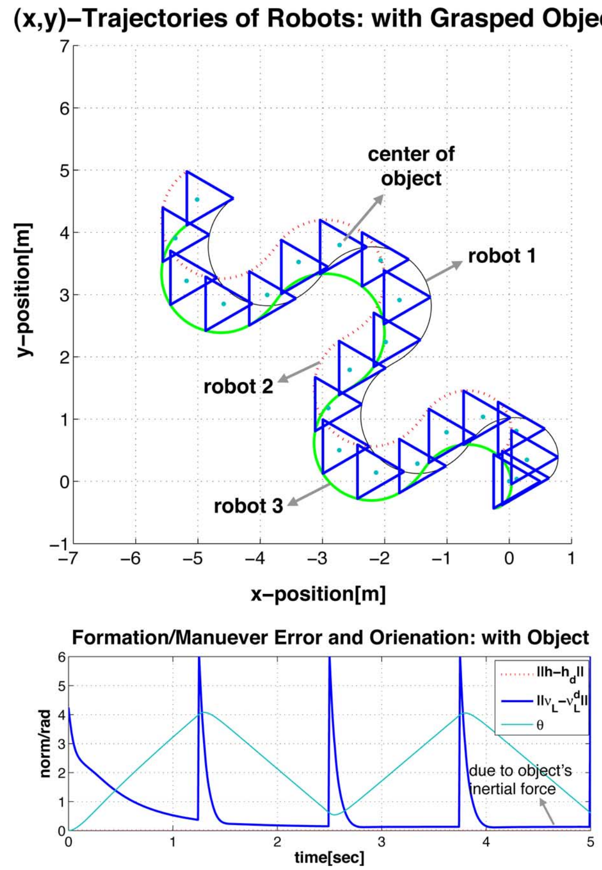

Fig. 8. Driving with formation keeping of three WMRs, with a cooperatively grasped inertial/flexible object.

again use $\mathcal{C}_\top := \mathcal{D}_\top^T$ for (38), since $\mathcal{D}_\top^T \mathcal{D}_\top = I$. The following is a corollary of Theorem 3.

Corollary 1: Consider the three WMRs with the control (u_L, u_c, u_E) in (34), and $h(q)$, $\varphi_E(h)$, $\nu_L^d(t)$ as defined earlier. Suppose $\theta_i(0) =: \theta(0)$ and $\dot{q}_i(0) = 0\ \forall i = 1, 2, 3$. Then, $\theta_i(t) =: \theta(t)\ \ \forall t \geq 0$, $(\dot{x}_i, \dot{y}_i, \dot{\theta}_i) \rightarrow (v_L^d\, \mathrm{c}\,\theta, v_L^d\, \mathrm{s}\,\theta, \dot{\theta}_d)$, and $h(q) \rightarrow h_d$ with $\varphi_E(t) \leq \varphi_E(0)\ \forall t \geq 0$.

Proof: First, note that the alignment error dynamics among θ_i are completely captured by $\mathcal{D}^c$. Also, from Theorem 3, if $\nu_c(0) = 0$, with u_c in (34), $\nu_c(t) = 0\ \forall t \geq 0$. Thus, we have, with $\theta_i(0) = \theta(0)$ and $\dot{\theta}_i(0) = 0$ (i.e., $\nu_c(0) = 0$) $\forall i = 1, 2, 3$, $\theta_i(t) = \theta(t)$ (from $\nu_c(t) = 0$) $\forall t \geq 0$. This also justifies our assumption of $\theta_1 = \theta_2 = \theta_3$ used for the derivation of NPD earlier. From Theorem 3, we also have $\nu_L \rightarrow [v_L^d; \dot{\theta}_d]$. This then implies that: 1) from the first column of $\mathcal{D}^\top \cap \Delta^\top$, $(\dot{x}_i, \dot{y}_i) \rightarrow v_L^d(\mathrm{c}\,\theta, \mathrm{s}\,\theta)$; and 2) from its second column, $\dot{\theta}_i \rightarrow \dot{\theta}_d$. Finally, with the given φ_E, we can write the condition (36) s.t.

$$S_E(\theta)\left[\frac{\partial \varphi_E}{\partial h}\right]^T = \begin{bmatrix} \dfrac{1}{m_1} + \dfrac{1}{m_2} & -\dfrac{1}{m_2} \\ -\dfrac{1}{m_2} & \dfrac{1}{m_2} + \dfrac{1}{m_3} \end{bmatrix} \begin{pmatrix} (h_1 - h_1^d)\,\mathrm{c}\,\theta + (h_2 - h_2^d)\,\mathrm{s}\,\theta \\ (h_4 - h_4^d)\,\mathrm{c}\,\theta + (h_5 - h_5^d)\,\mathrm{s}\,\theta \end{pmatrix} \tag{44}$$

where h_i and h_i^d are the ith component of $h, h_d \in \Re^6$. Here, θ will oscillate due to $\dot{\theta} \rightarrow \dot{\theta}_d$, and the matrix in the second line is invertible. Thus, similar to Theorem 3, $h_i \rightarrow h_i^d$ for $i = 1, 2, 4, 5$. This then implies $h(q) \rightarrow h_d$, since $h_i(t) = h_i^d\ \forall t \geq 0$ for $i = 3, 6$ (from $\theta_1 = \theta_2 = \theta_3\ \forall t \geq 0$). ■

Simulation results are shown in Figs. 7–10, each consisting of: 1) snapshots of the robots' (x, y)-motion and their triangular/line formations (vertexes corresponding to each robot); and 2) plots of the formation error $\|h - h_d\|$, θ-motion, and/or maneuver error $\|\nu_L - \nu_L^d\|$.

First, in Fig. 7, the three WMRs start with the desired formation shape with $h(0) = h_d$, while the desired maneuver (i.e., $\dot{\theta}_d$) switches about every 1.25 s to produce "snake-like" motion. Even with these maneuver switchings, due to the formation-maneuver decoupling and $\varphi_E(t) \leq \varphi_E(0) = 0$ (i.e., $h(t) = h_d$ $\forall t \geq 0$), the desired formation shape is maintained all the time. In Fig. 8, we insert an inertial/deformable object (with green dots represent its center point within the triangular formation) to emulate *fixtureless* cooperative manipulation. With the cancellation δ_E in (34), the robots can still keep the desired formation shape and maintain the grasping of the object, even with the maneuver switchings. Here, we omit the cancellation δ_L in (34) so that the object's behavior can affect the robots' motion (e.g., nonzero $\|\nu_L - \nu_L^d\|$ between switchings in Fig. 8). This would be

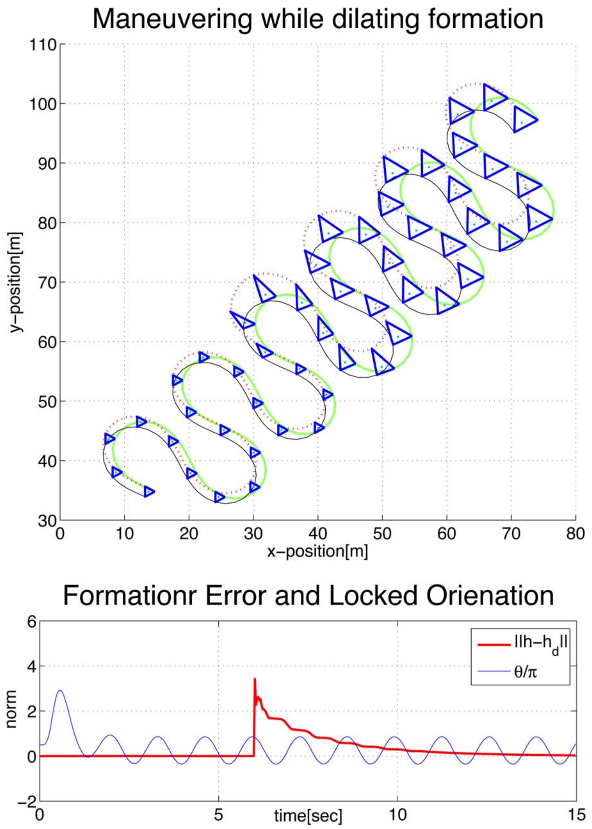

Fig. 9. Maneuvering with formation dilation, where light blue dots represent locked system's (x, y) position obtained via integration of $(\mathcal{D}^\top \cap \Delta^\top)\nu_L$.

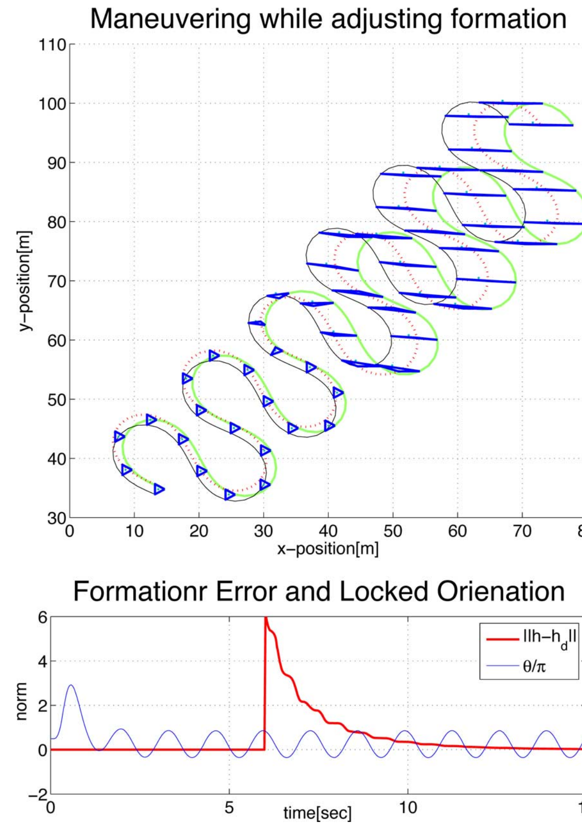

Fig. 10. Maneuvering with formation change.

useful for teleoperation, where humans often want to perceive the grasped object's behavior/inertia [21].

On the other hand, in Figs. 9 and 10, h_d switches (around 6 s) to change the robots' triangular formation shape to a larger one (see Fig. 9) or to a line formation (see Fig. 10). For both of them, with the θ-motion satisfying (44), the formation shape converges to the desired one. Moreover, due to the formation-maneuver decoupling, the formation switchings do not affect the maneuver behavior (e.g., same θ-motion in Figs. 9 and 10).

Remark 1: For the WMRs with the linear h, if $m_i = m_j, I_i = I_j$, $d_i = d_j$ (or $d_i = 0$), we will have strong decomposability with $\mathcal{D}^c$ contained in $\Delta^\perp$, and $Q_{\alpha\beta} = 0$ for (27), since $Q_{\alpha\beta}$ contains terms like $m_1 d_1 I_2 - m_2 d_2 I_1$ [12].

VII. Summary and Future Works

In this paper, we propose nonholonomic passive decomposition (NPD), with which we can decompose the Lagrange–D'Alembert dynamics of multiple (or a single) nonholonomic mechanical systems into 1) shape system, representing the formation aspect (e.g., cooperative grasping); 2) locked system, describing the maneuver aspect (e.g., motion of the grasped object); 3) quotient system, perturbing both the formation and maneuver aspects (and vanishing for certain cases); and 4) inertia-induced conservative coupling among them. All the locked/shape/quotient systems individually inherit Lagrangian-like structure and passivity. We also introduce notions of formation/maneuver d-controllability to address restrictions imposed by the nonholonomic constraint and provide passivity-based control design examples for simultaneous/separate control of the formation and maneuver aspects. Some numerical examples are discussed to illustrate theory. Extension to kinematic nonholonomic systems is also presented.

Some directions for future research include: 1) underactuated control design in $\mathcal{C}^\top$; 2) relation between excitation condition (36) and formation d-controllability (31); 3) role of symmetry of nonholonomic mechanical systems and its utilization for passive decomposition (see [22] for a preliminary result in this direction); and 4) coordinate-invariant/differential-geometric formulation of NPD, similar to that of standard passive decomposition [2], [3].

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