

# Backstepping Control of Quadrotor-Type UAVs and Its Application to Teleoperation over the Internet

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**Abstract.** We propose a novel exponentially-stable backstepping trajectory tracking control law for unmanned aerial vehicles (UAVs), consisting of translational dynamics and attitude kinematics on SE(3), with one thrust force and two angular rates along three orthogonal axes as control inputs. Its application to the recently-proposed UAV Internet teleoperation control architecture [1] is explained, with a new dynamic-extension filter to avoid discontinuity in the control implementation. Experimental results using a real indoor quadrotor-type UAV are also presented to show the efficacy of the proposed theory.

## 1 Introduction

Unmanned aerial vehicles (UAVs) are promising to achieve many useful applications with the cost associated to the onboard human pilots removed: landscape survey, entertainment and games, surveillance/reconnaissance, remote repair, and precise unmanned attack, to name a few. In particular, quadrotor-type UAVs have recently received much attention, due to its agility, (relative) easiness of control, and availability [2]. Teleoperation of such quadrotor UAVs would even further expand the application horizon of this versatile flying robotic platform [1].

In the first half of this paper, we propose a novel exponentially-stable backstepping trajectory tracking control law for “mixed” quadrotor-type UAVs, which can be modeled as a combination of translational dynamics in E(3) and attitude kinematics in SO(3), with the thrust force  $\lambda \in \mathfrak{R}$  and the angular rates  $w \in \mathfrak{R}^3$  as the control input. This system is under-actuated (i.e., only 4 control inputs for 6-dimensional SE(3) motion). To address this under-actuation issue, we utilize the backstepping technique on top of the passivity property of the UAV’s translation dynamics [3].

Here, we focus on the “mixed” UAVs (i.e., with translation dynamics and attitude kinematics), since many commercially available UAVs, including our laboratory system, Asctec Hummingbird<sup>®</sup>, being shipped with a high-performance low-level attitude control-loop already in place, allow us to directly send angular rate command. Due to this, our control law is much simpler (thus, easier to implement) than other backstepping control laws, that are derived for “dynamic” UAVs (i.e., with translation and attitude dynamics [4,5,6]). Our backstepping control also: 1) exploits the geometry of SE(3), thus,

is free from the singularity due to the  $SO(3)$  parameterization (e.g., [7,8]); 2) has transparent control parameters (e.g., damping/spring gains; convergence time constant, etc), thus, can be tuned intuitively (cf. [9,10]); and 3) is flexible in the sense that other kinds of control laws designed for the point mass dynamics would be incorporated into our backstepping framework (e.g. path following, distributed coordination [11]).

We then apply this backstepping trajectory tracking control to the recently proposed UAV Internet teleoperation control architecture of [1]. In particular, we utilize our backstepping control to drive the real quadrotor UAV to follow the trajectory of the (kinematic) Cartesian virtual point (VP), which is teleoperated by a remote human user over the Internet. This UAV-VP coordination problem was only alluded in [1] and, in this paper, we fill that gap. More specifically, 1) we propose a dynamic-extension filter to circumvent the problem of using (potentially discontinuous/unbounded) high-order derivatives of the VP's position in our backstepping trajectory tracking control for the UAV-VP coordination; and 2) provide complete stability/collision-avoidance proof of the combination of the dynamic-extension filter and the VP's kinematic evolution, with the (bounded, yet, arbitrary) master set-position command received from the Internet. Similar to [1], we also apply passive set-position modulation (PSPM [12]) to passify the master side with unreliable/imperfect Internet (with the master position then guaranteed to be bounded), and prove the combination of the master-passivity and slave-stability over the Internet. We also present some pilot experimental results, obtained with Asctec Hummingbird<sup>®</sup>, VICON Bonita motion capture system<sup>®</sup>, and Sensable PHANToM Omni<sup>®</sup>, for the trajectory tracking and the Internet teleoperation.

The rest of this paper is organized as follows. Sec. 2 presents the modeling of the “mixed” quadrotor-type UAV. Our backstepping control is derived and detailed, and its robustness analyzed in Sec. 3. We apply it to UAV teleoperation over the Internet in Sec. 4. Some concluding remarks are given in Sec. 5.

## 2 Underactuated Quadrotor-Type UAVs

We consider the following quadrotor-type UAV, evolving on  $SE(3)$  according to the translation dynamics and attitude kinematics [9]:

$$m\ddot{x} = -\lambda Re_3 + mge_3 + \delta \quad (1)$$

$$\dot{R} = RS(w) \quad (2)$$

where  $m > 0$  is the mass,  $x \in \mathfrak{R}^3$  is the Cartesian position w.r.t. the NED (north-east-down) inertial frame with  $e_3$  representing its down-direction,  $\lambda \in \mathfrak{R}$  is the thrust along the body-frame down direction,  $\delta \in \mathfrak{R}^3$  is the Cartesian disturbance,  $R \in SO(3)$  is the rotational matrix describing the body NED frame of UAV w.r.t. the inertial NED frame,  $w := [w_1, w_2, w_3] \in \mathfrak{R}^3$  is the angular velocities of the body frame relative to the inertial frame expressed in the body frame,  $g$  is the gravitational constant, and  $S(\star) : \mathfrak{R}^3 \rightarrow so(3)$  is the skew-symmetric operator defined s.t. for  $a, b \in \mathfrak{R}^3$ ,  $S(a)b = a \times b$ .

The control inputs for (1)-(2) are the thrust force  $\lambda \in \mathfrak{R}$  and the angular rates  $w \in \mathfrak{R}^3$ . This “mixed” UAV (1)-(2) can capture many commercially available UAVs shipped with a manufacturer's low-level attitude control servo-loop implemented (e.g. Asctec

Hummingbird®). These control inputs  $(\lambda, w)$  obtained for (1)-(2) can also be often applied to “dynamic” UVAs as well (i.e., with translation and attitude dynamics). This is because, with the UAVs’ attitude dynamics typically fully-actuated and passive, it is straightforward to design angular torque control to track this target angular rate  $w$ . Note also that the UAV (1)-(2) is under-actuated, with only 4-DOF control for 6-DOF SE(3) motion.

In the next Sec. 3, we utilize backstepping technique [3] to overcome this under-actuation of (1)-(2) and to drive its Cartesian position  $x(t)$  to track a smooth desired trajectory  $x_d(t)$ . This backstepping trajectory tracking control will then be combined to the teleoperation control framework of [1] and analyzed together in Sec. 4.

### 3 Backstepping Trajectory Tracking Control of UAVs

Following [11], we first design the desired trajectory tracking control  $v \in \mathfrak{R}^3$  s.t.

$$\lambda Re_3 = \underbrace{-m\ddot{x}_d + mge_3 + b\dot{e} + ke + v_e}_{=:v} \quad (3)$$

where  $x_d(t) \in \mathfrak{R}^3$  is the smooth desired trajectory with all  $\dot{x}_d(t), \ddot{x}_d(t), \ddot{\ddot{x}}_d(t)$  bounded,  $e(t) := x(t) - x_d(t)$  is the tracking error,  $b, k > 0$  are damping/spring gains, and  $v_e \in \mathfrak{R}^3$  is the control generation error due to the under-actuation of (1)-(2). In general,  $v_e \neq 0$ , since the last column of  $R$  (i.e.  $Re_3$ ) is not necessarily aligned with the desired control  $v$ . Here, to derive our “nominal” control, let us temporarily assume  $\delta = 0$  in (1). Effect of non-zero disturbance  $\delta$  will be reported in a future publication.

We can then write the closed-loop dynamics

$$m\ddot{e} + b\dot{e} + ke = -v_e \quad (4)$$

for which we define

$$V_1 := \frac{1}{2}m\dot{e}^T \dot{e} + m\epsilon e^T \dot{e} + \frac{1}{2}(k + \epsilon b)e^T e$$

where  $\epsilon > 0$  is a constant to be chosen below. Differentiating this  $V_1$  with (4), we then have

$$\dot{V}_1 = -(b - m\epsilon)\dot{e}^T \dot{e} - \epsilon ke^T e - (\dot{e} + \epsilon e)^T v_e = -\zeta^T Q \zeta - (\dot{e} + \epsilon e)^T v_e \quad (5)$$

where  $\zeta := [\dot{e}; e]^T \in \mathfrak{R}^6$  and

$$Q := \begin{bmatrix} b - \epsilon m & 0 \\ 0 & \epsilon k \end{bmatrix} \otimes I_3, \quad P := \begin{bmatrix} m & \epsilon m \\ \epsilon m & k + \epsilon b \end{bmatrix} \otimes I_3$$

with  $V_1 = \zeta^T P \zeta / 2$ , where  $\otimes$  is the Kronecker product and  $I_3 \in \mathfrak{R}^{3 \times 3}$  is the identity. Both  $P$  and  $Q$  will then be positive-definite (i.e.  $P \succ 0$  and  $Q \succ 0$ ), if we choose  $\epsilon > 0$  s.t.

$$0 < \epsilon < b/m \quad (6)$$

with the condition for  $Q \succ 0$  always implied by that for  $P \succ 0$  with  $k > 0$ .

If  $v_e = 0$  in (5), we would have  $(\dot{e}, e) \rightarrow 0$  exponentially. To address the term with  $v_e$  in (5), let us augment  $V_1$  s.t.

$$V_2 = V_1 + \frac{1}{2\gamma} v_e^T v_e$$

where  $\gamma > 0$  is a constant. Differentiating this  $V_2$ , we then have

$$\dot{V}_2 = -\frac{b}{2} \dot{e}^T \dot{e} - \epsilon k e^T e + \frac{1}{\gamma} v_e^T (\dot{v}_e - \gamma(\dot{e} + \epsilon e))$$

which suggests the backstepping update law for  $\dot{v}_e$  s.t.

$$\dot{v}_e = \gamma(\dot{e} + \epsilon e) - \alpha v_e \quad (7)$$

so that, with  $\alpha > 0$ , we can obtain

$$\dot{V}_2 = -\frac{b}{2} \dot{e}^T \dot{e} - \epsilon k e^T e - \frac{\alpha}{\gamma} v_e^T v_e$$

implying exponential convergence of  $e, \dot{e}$  and  $v_e$ . Here, we assume  $\epsilon$  is chosen according to (16), therefore, known.

Control design for  $(\lambda, w)$  is in fact embedded in the update law (7) and needs to be decoded. For this, using (3), we rewrite (7) s.t.

$$[(\dot{\lambda} + \alpha\lambda)R + \lambda RS(w)]e_3 = \dot{v} + \alpha v + \gamma(\dot{e} + \epsilon e) =: \bar{v} \quad (8)$$

which can be reorganized as

$$\begin{pmatrix} \lambda w_2 \\ -\lambda w_1 \\ \dot{\lambda} + \alpha\lambda \end{pmatrix} = R^T \bar{v} \quad (9)$$

where  $\bar{v}_i \in \mathfrak{R}$  is the  $i$ -th element of  $\bar{v} \in \mathfrak{R}^3$ , and  $\dot{v}$  for  $\bar{v}$  can be computed by

$$\dot{v} = -m\ddot{x}_d - b\dot{x}_d + k\dot{e} + \frac{b}{m}(-\lambda R e_3 + m g e_3) \quad (10)$$

to avoid the usage of  $\ddot{x}$ . We can then compute the control inputs  $(\lambda, w_1, w_2)$  from (9): 1) compute  $w_2, w_1$  from the first and second rows of (9) as long as  $\lambda \neq 0$ ; and 2) update  $\lambda$  by solving the differential equation in the last row of (9).

Assuming  $\lambda \neq 0$  (to obtain  $w_1, w_2$ ) is typical for other UAV controls as well (e.g., [4,5,6]), which anyway seems to unlikely happen in practice (e.g., no free fall). Note from (9) that, for the Cartesian position control, we only need  $\lambda, w_1$  and  $w_2$ , not  $w_3$ . This is again typical for UAVs control [4,5,6], and we may simply set  $w_3 = 0$  or use it for other purpose (e.g., coordinated observation). Since the control parameters  $b, k, \alpha$  have clear physical meanings, their tuning can be done intuitively for our control (unlike, e.g., [9]). Due to considering only attitude kinematics, the control decoding equation (9) is also substantially simpler than that in [5,6], which is based on the attitude dynamics. The relation (9) also shows that any (smooth) desired control  $v$  can be incorporated into our backstepping control design, as long as it produces a relation similar to (9) and its computation is implementable similar to  $\dot{v}$  in (10) here. We now summarize some key properties of our backstepping trajectory tracking control.

**Theorem 1.** Consider the UAV (1)-(2) under the backstepping control (8)-(9) with  $w_3$  and  $\dot{x}_d, \ddot{x}_d, \ddot{\ddot{x}}_d$  all being bounded. Suppose that  $\exists \varepsilon_\lambda > 0$  s.t.  $\lambda(t) \geq \varepsilon_\lambda \forall t \geq 0$ . Then,  $(\dot{e}, e, v_e) \rightarrow 0$  exponentially; and  $(\dot{x}, \ddot{x}, \lambda, w)$  are bounded.

## 4 Application to UAV Teleoperation over the Internet

In this section, we show how the backstepping control, designed/analyzed in Sec. 3, can be used for the UAV Internet teleoperation within the recently proposed framework of [1]. Following [1], consider a (first-order kinematic) Cartesian virtual point (VP), evolving according to

$$\dot{p} = \eta q(k) - \frac{\partial \varphi_o^T}{\partial p}$$

where  $p \in \mathfrak{R}^3$  is the VP's position,  $q(k) \in \mathfrak{R}^3$  is the master device's position  $q(t) \in \mathfrak{R}^3$  received via the Internet at the (slave) reception time  $t_k^s$ ,  $\eta > 0$  is to match different scales between  $q(t)$  and  $\dot{p}$ , and  $\varphi_o(\|p - p_o\|)$  is the obstacle avoidance potential, which produces a repulsive force when  $p$  approaches the obstacle at  $p_o$ . The command  $\eta q(k)$  in (11) enables the user to tele-control the VP's velocity  $\dot{p}$  by the master device's position  $q(t)$ , addressing the problem of master-slave kinematic dissimilarity (i.e. stationary master with bounded workspace; mobile VP with unbounded workspace [13,14]). The kinematic VP is also chosen here as in [1], since it greatly simplifies the stability and collision avoidance analysis as shown below.

The UAV position (1)-(2) then needs to track this VP's position with  $(x, \dot{x}) \rightarrow (p, \dot{p})$ . To apply our backstepping control (9) for this trajectory tracking, as can be seen from (10), we need to compute not only  $p, \dot{p}$ , but also  $\ddot{p}$  and  $\ddot{\ddot{p}}$ . This requirement of higher-order derivatives of the desired trajectory is in fact true for most of the UAV trajectory tracking control schemes (e.g., [4,5,6,9,10]). Yet, since  $q(k)$  switches at each data reception time  $t_k^s$  from the (discrete-time) Internet, its time-derivative is not well-defined, and so are  $\dot{p}, \ddot{p}$ .

To remedy this problem, we adopt a dynamic-extension filter and use its (continuous-time/smooth) output  $\bar{q}(t)$  to control the VP instead of  $q(k)$ . More specifically, we simulate the VP's position s.t.

$$\dot{p} = \eta \bar{q}(t) - \frac{\partial \varphi_o^T}{\partial p} \quad (11)$$

where  $\bar{q}(t)$  is defined from

$$\ddot{\bar{q}}(t) + 2b'\dot{\bar{q}}(t) + k'\bar{q}(t) = k'q(k) \quad (12)$$

with  $b', k'$  chosen s.t. the second-order filter is critically damped. Here, note that, if  $q(k)$  is bounded,  $\bar{q}(t), \dot{\bar{q}}(t), \ddot{\bar{q}}(t)$  are all bounded and well-defined. Then, we can compute  $\ddot{p}$  and  $\ddot{\ddot{p}}$  s.t., from (11),

$$\ddot{p} = \eta \dot{\bar{q}}(t) - H_{\varphi_o}(p)\dot{p} \quad (13)$$

$$\ddot{\ddot{p}} = \eta \ddot{\bar{q}}(t) - H_{\varphi_o}(p)\ddot{p} - \frac{dH_{\varphi_o}(p)}{dt}\dot{p} \quad (14)$$

where  $H_{\varphi_o}(p) := \left[ \frac{\partial^2 \varphi_o}{\partial p_i \partial p_j} \right] \in \mathfrak{R}^{3 \times 3}$  is the Hessian of  $\varphi_o$ .

Now, to analyze the combined stability and obstacle avoidance of the VP dynamics with the dynamic extension filter (12), define

$$V_p(p, \bar{q}, \dot{\bar{q}}) := \varphi_o(p) + \frac{1}{2} \|\dot{\bar{q}}\|^2 + \bar{\varepsilon} \dot{\bar{q}}^T \bar{q} + \frac{1}{2} (k' + \bar{\varepsilon} b') \|\bar{q}\|^2$$

where  $\bar{\varepsilon} > 0$  is small to make the part of  $V_p$  with  $\bar{q}$  and  $\dot{\bar{q}}$  to be positive-definite. Then, we can show that, from (11) (with  $q(k)$  replaced by  $\bar{q}(t)$ ) and (12),

$$\frac{dV_p}{dt} = -\xi^T \bar{Q} \xi - u^T \xi \leq -\underline{\sigma}[\bar{Q}] \cdot \|\xi\|^2 + \|u\| \cdot \|\xi\| \quad (15)$$

where  $\underline{\sigma}[\bar{Q}]$  is the minimum singular value of  $\bar{Q}$ , and

$$\bar{Q} := \begin{bmatrix} 1 & -\frac{\eta}{2} & 0 \\ -\frac{\eta}{2} & \bar{\varepsilon} k'^2 & 0 \\ 0 & 0 & b' - \bar{\varepsilon} \end{bmatrix} \otimes I_3$$

with  $\xi := [\partial \varphi_o^T / \partial p; \bar{q}; \dot{\bar{q}}]$ , and  $u := k' q(k) \cdot [0; \bar{\varepsilon}; 1] \otimes I_3$ .

Thus, if we choose  $\bar{\varepsilon} > 0$  to satisfy the following conditions,  $V_p$  will be positive-definite w.r.t.  $\dot{\bar{q}}, \bar{q}$  and also  $Q \succ 0$ :

$$\bar{\varepsilon}^2 - b' \bar{\varepsilon} - k' < 0, \quad \bar{\varepsilon} < b', \quad \bar{\varepsilon} k' > \eta^2 / 2$$

which can be further simplified as

$$\eta^2 / (2k') < \bar{\varepsilon} < b' \quad (16)$$

since the second condition in the first set of the conditions is implied by (16). The inequality (15) then implies that, similar to the case of ultimate boundedness, if  $\|\xi\| \geq \frac{\|u\|}{\underline{\sigma}[\bar{Q}]}$ , we will have  $\dot{V}_p \leq 0$ . Based on this observation, we have the following Prop. 1. For that, we assume that  $\varphi_o$  is constructed s.t. 1) there exists a large enough  $\bar{M} > 0$  s.t.  $V(t) \leq \bar{M}$  implies no collision with obstacle  $p_o$ ; and 2) if  $\varphi_o(\|p - p_o\|)$  gets very large, so does  $\|\partial \varphi_o / \partial p\|$ .

**Proposition 1.** *Suppose  $q(k)$  is bounded, i.e.,  $\exists q_{\max} \geq 0$  s.t.  $q_{\max} \geq \|q(k)\|, \forall k \geq 0$ . Suppose further that, if  $\varphi_o(\|p - p_o\|) \geq \bar{M}$ ,*

$$\left\| \frac{\partial \varphi_o}{\partial p} \right\| \geq \frac{k' q_{\max} \sqrt{1 + \bar{\varepsilon}^2}}{\underline{\sigma}[\bar{Q}]} \quad (17)$$

*Then,  $\varphi_o \leq \bar{M} \forall t \geq 0$  and  $\bar{q}, \dot{\bar{q}}, \ddot{\bar{q}}$  are bounded. Suppose further that  $\partial \varphi_o^2 / \partial p_i \partial p_i$  and  $\partial \varphi_o^3 / \partial p_i \partial p_i \partial p_k$  are bounded if  $\varphi_o \leq \bar{M}$ . Then,  $\dot{p}, \ddot{p}, \ddot{\ddot{p}}$  are all bounded.*

With  $\dot{p}, \ddot{p}, \ddot{\ddot{p}}$  all well-defined, the backstepping control of Sec. 3 can then robustly enable the UAV to track the trajectory of the VP. Here, we assume the obstacle perception potential  $\varphi_o$  is designed s.t. it rapidly increases when the VP  $p$  approaches very close to the obstacle  $p_o$  to produce high repulsive force to prevent collision; while gradually

converge to zero as  $\|p - p_o\| \rightarrow 0$  so that the effect of obstacles can smoothly emerge when they gets close to the VP. An example of such  $\varphi_o$  is

We feed back this obstacle avoidance force along with the UAV's velocity to the human users so that they can perceive the presence of the obstacle over the Internet and/or the state of the UAV. For this, we design the haptic feedback signal  $y(t) \in \mathfrak{R}^3$  to be sent to the master, s.t.

$$y(t) := \frac{1}{\eta} \left( \dot{x} + \frac{\partial \varphi_o^T}{\partial p} \right) \quad (18)$$

where the two terms,  $\dot{x}/\eta$  and  $(1/\eta)\partial\varphi_o^T/\partial p$ , are typically complementary, i.e., during the free cruise flying,  $\partial\varphi_o^T/\partial p \approx 0$ , whereas for the contact with the obstacle,  $\dot{x} \approx 0$ .

This  $y(t)$  is then sent to the master over the Internet. Let us denote by  $y(k)$  its reception by the master side over the Internet at the (master) reception time  $t_k$ . We incorporate this  $y(k)$  into the PD-type teleoperation control  $\tau$  for the master device s.t.

$$\tau(t) := -B\dot{q} - K_1q - K(q - \bar{y}(k)) \quad (19)$$

for  $t \in [t_k, t_{k+1})$ , where  $B, K_1, K \succ 0$  are diagonal gain matrices, and  $\bar{y}(k)$  is the PSPM-modulation of  $y(k)$  (to be defined below). Here,  $K_1$  is included to provide haptic feedback of  $y(t)$  (i.e. perception of UAVs velocities or presence of obstacles) while  $K$  attempts  $\|q(t) - \bar{y}(k)\| \rightarrow 0$ .  $B$  is used to avoid oscillatory behavior.

If we use  $y(k)$  directly received from the unreliable Internet for (19) instead of  $\bar{y}(k)$ , the PD-coupling (19) can become unstable. To address this problem, as proposed in [1], we also adopt here passive set-position modulation (PSPM) [12], which is more flexible (e.g., passive feedback of  $y(k)$ ) and less conservative (i.e., selective activation of passifying action only necessary) than conventional time-invariant passivity-enforcing frameworks (e.g., PD-based or wave-based techniques). More precisely, at each  $t_k$ ,  $\bar{y}(k)$  in (19) is computed s.t.

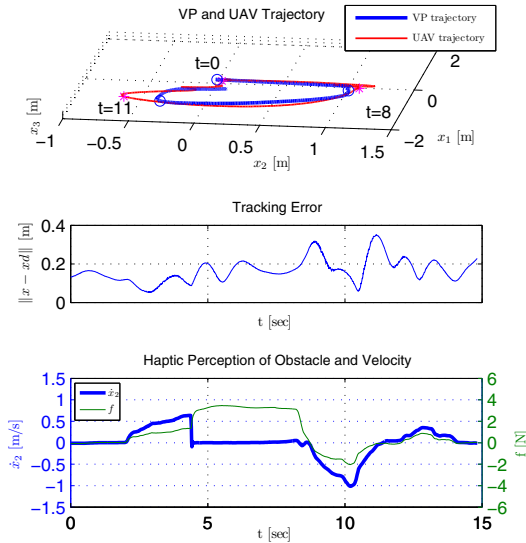
$$\begin{aligned} & \min_{\bar{y}(k)} \|y(k) - \bar{y}(k)\| \\ & \text{subj. } E(k) = E(k-1) + D_{\min}(k-1) - \Delta\bar{P}(k) \geq 0 \end{aligned}$$

where the second line is to enforce passivity, which would likely be violated if we directly utilize (switching)  $y(k)$  in (19) without any remedy. Here,  $E(k) \geq 0$  is the virtual energy reservoir;  $\Delta\bar{P}(k) := \|q(t_k) - \bar{y}(k)\|_K^2/2 - \|q(t_k) - \bar{y}(k-1)\|_K^2/2$  with  $\|x\|_A^2 := x^T A x$ ; and  $D_{\min}(k) := \frac{1}{t_{k+1} - t_k} \sum_{i=1}^3 b_i (\bar{q}_i(k) - \underline{q}_i(k))^2$ , with  $b_i > 0$  being the  $i^{\text{th}}$  diagonal element of  $B$ ,  $q_i$  the  $i^{\text{th}}$  element of  $q$ , and  $\bar{q}_i(k)/\underline{q}_i(k)$  the max/min of  $q_i(t)$  during  $[t_k, t_{k+1})$ ,  $i = 1, 2, 3$ . Note that this PSPM is implemented only for the master side. Also, since the human operator usually keeps injecting energy into the master,  $E(k)$  may keep increasing as well. To avoid excessive energy accumulation in  $E(k)$ , we ceil off  $E(k)$ , by discarding any energy over a certain threshold  $\bar{E}$ . See [12] for more details on PSPM.

**Theorem 2.** 1) The master device with PSPM-modulated control (19) is closed-loop passive:  $\exists c_1 \in \mathfrak{R}$  s.t.,  $\int_0^T f^T \dot{q} dt \geq -c_1^2, \forall T \geq 0$ , where  $f, \dot{q} \in \mathfrak{R}^3$  are the human force and velocity. Moreover, if the human user is passive (i.e.  $\exists c_2 \in \mathfrak{R}$  s.t.,  $\int_0^T f^T \dot{q} dt \leq c_2^2, \forall T \geq 0$ ), the closed-loop VPs teleoperation system is stable, with  $\dot{q}, q, q - \bar{y}(k)$ , and  $\dot{p}_i$  all bounded.

2) Suppose further that  $\ddot{q}, \dot{q} \rightarrow 0, E(k) > 0 \forall k \geq 0$ , and  $(x, \dot{x}) = (p, \dot{p})$ . Then, (a) if  $\partial \phi_o / \partial p = 0$  (e.g. no obstacles),  $f(t) \rightarrow \frac{K_1}{\eta} \dot{x}_i$  (i.e. UAV velocity perception); or (b) if  $\dot{x} = 0$  (e.g. stopped by obstacles),  $f(t) \rightarrow \frac{K_1}{\eta} \partial \phi_o / \partial p$  (i.e. collective obstacle perception).

Notice the flexibility in designing/using the haptic feedback  $y(t)$  (18) provided by PSPM (e.g. other forms of  $y(t)$  can be used without jeopardizing passivity). Such a flexibility is usually not achievable by other passivity-based schemes (e.g. wave/PD). The item 1) of Th. 2 and Prop. 1 essentially establishes master-passivity/slave-stability of our closed-loop teleoperation system, which, we believe, is less conservative and more suitable for UAV teleoperation than conventional master-slave (energetic) passivity (e.g., humans need to continuously overcome wind drag or other physical dissipations of UAV - see [15]). Experimental results are shown in Fig. 1, where we can see that: 1) obstacle avoidance is activated and prevents the VP and UAV to collide with the obstacle (4-8 sec.); and 2) the human can haptically perceived the velocity of the UAV (2-4, 8-15 sec.) or the presence of the obstacle (4-8 sec.), with  $y(k)$  (18) complementarily switching between these two modes on its own.



**Fig. 1.** Teleoperation with haptic perception of obstacle avoidance and flying velocity



## 5 Conclusions

In this paper, we propose an exponentially-stable backstepping trajectory tracking control law for the quadrotor-type UAV with  $E(3)$  dynamics and  $SO(3)$  kinematics. Robustness analysis is given. Its application to the UAV Internet teleoperation according to the framework of [1] is explained. Some pilot experiments are performed to validate the theory.

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