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Brief paper

Distributed backstepping control of multiple thrust-propelled vehicles on a balanced graph☆

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ABSTRACT

We propose novel exponentially-converging distributed flocking and formation-centroid backstepping control frameworks for multiple under-actuated thrust-propelled vehicles in SE(3), each consisting of the under-actuated Cartesian dynamics on E(3) with one-dimensional thrust-force input and the fully-actuated attitude kinematics on SO(3) with angular-rates inputs; and evolving on a strongly-connected balanced information graph G. Simulations are also performed to illustrate the theory.

1. Introduction

Many autonomous robotic vehicles share the following property (Aguiar & Hespanha, 2007; Hua, Hamel, Morin, & Samson, 2009): they are six-degree-of-freedom (DOF) dynamic systems evolving on $SE(3) = E(3) \times SO(3)$, with their attitude dynamics in SO(3) fully-actuated, yet, their Cartesian dynamics in E(3) under-actuated with only one-DOF thrust-actuation, whose direction is fixed to the vehicle's body-frame. Some examples include quadrotor unmanned aerial vehicles (UAVs) (Frazzoli, Dahleh, & Feron, 2000; Hua et al., 2009), and underwater/sea-surface vessels with symmetric configuration (Børhaug, Pavlov, Panteley, & Pettersen, 2011; Ghabcheloo et al., 2009). Following (Hua et al., 2009), we call such systems thrust-propelled vehicles (TPVs).

In this paper, we propose a novel distributed backstepping control framework for these *under-actuated* TPVs on a time-invariant balanced (yet, otherwise directed) information graph G. We particularly solve the following two problems: (1) flocking (Lee & Spong, 2007; Tanner, Jadbabaie, & Pappas, 2007), i.e., the TPVs cooperatively attain a desired formation shape with all their velocities converging to a common unspecified constant; and (2) formation-centroid control (Lee, 2010; Lee & Li, 2007), i.e., same as flocking, yet, in this case, TPVs' centroid position (or velocity) converges to a desired timed-trajectory.

The unique challenge here is the TPVs' under-actuation, which renders many flocking/consensus results, derived for simple agents with full-actuation, inapplicable (e.g., Lee & Spong, 2007; Olfati-Saber & Murray, 2004; Ren, 2007; Ren, Beard, & Atkins, 2007; Su, Wang, & Lin, 2009; Tanner et al., 2007; Xiao & Wang, 2008). To overcome this challenge, we extend the passive decomposition of Lee and Spong (2007) to explicitly incorporate the topology of G, with which we can not only (still) decompose the centroid and formation dynamics of multiple TPVs and design a suitable "lumped" backstepping control (Sepulchre, Jankovic, & Kokotovic, 1997) to address TPVs' under-actuation; but also reveal that this designed backstepping control, albeit seemingly fully-centralized, is in fact decentralizable over a balanced graph G. Our proposed control is also exponentially-stable, thus, robust against any bounded disturbance (e.g., model uncertainty or aerodynamic force (Hua et al., 2009)); and the centroid and formation controls do not interfere with each other, except exponentially-decaying crosstalk between them.

Backstepping is widely used for a *single* under-actuated system (e.g., for TPV (Aguiar & Hespanha, 2007; Frazzoli et al., 2000; Hua et al., 2009)), yet, only rarely for multiple of them over a graph G, due to the difficulty in finding a suitable (nominal) Lyapunov function for backstepping design (e.g., spring energy not defined over directed G), or, perhaps more significantly, in decentralizing

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the obtained control over G, particularly when G is directed. Some representative results are: (1) Cui, Ge, How, and Choo (2010); Ghabcheloo et al. (2009), where a fictitious abstraction layer (e.g., virtual target (Ghabcheloo et al., 2009)) is added to convert the coordination problem of (real) under-actuated vehicles into that of fictitious states and backstepping is used to drive each vehicle to track its own abstraction; and (2) Listmann and Woolsey (2009), where a backstepping control is designed to synchronize multiple chained-form systems embedded in the Euclidean space, thus, not applicable to TPVs evolving on non-Euclidean SE(3).

Some other notable results, relevant to this paper, yet, not relying on backstepping, are as follows: (1) distributed control of under-actuated marine vehicles on a graph is addressed in (Børhaug et al., 2011; Dong, 2010), yet, limited only for planar systems on SE(2), straightline tracking (Børhaug et al., 2011) or identical agents (Dong, 2010); (2) although with some similarity with our result in that the behavior of multiple systems is decomposed using Jacobi shape coordinates (Zhang, 2010), the scheme of Yang and Zhang (2010) is centralized and limited to SE(2); and (3) the results of Abdessameud and Tayebi (2009, 2010), perhaps most closely-related to our paper, consider the same distributed control problem of TPVs, yet, with more restrictions (i.e., require TPVs' acceleration $\|\ddot{x}(t)\| < g$ (e.g., aggressive sway impossible); undirected graph G; quaternion instead of SO(3)).

In contrast, our result here fully exploits the geometric structure of non-Euclidean SE(3) = E(3) × SO(3) of the TPVs; does not require any intermediate fictitious abstraction; is distributable over any balanced graph G; and only requires $\ddot{x}(t) \neq [0, 0, g]^T$ (i.e., no free fall - similar to Frazzoli et al. (2000); Hua et al. (2009)).

The rest of the paper is organized as follows. Preliminary materials are given in Section 2. Distributed backstepping flocking and formation-centroid controls of multiple TPVs on a balanced and strongly-connected graph are respectively designed and simulated in Sections 3 and 4. Concluding remarks are given in Section 5.

2. Preliminary

We consider a team of N "mixed"[2] thrust-propelled vehicles (TPVs), each consisting of the Cartesian dynamics in E(3) and the attitude kinematics in SO(3) (Hua et al., 2009), i.e., for the ith agent,

$$m_i\ddot{x}_i = -\lambda_i R_i e_3 + m_i g e_3 + \delta_i \quad (1)$$

$$\dot{R}_i = R_i S(w_i) \quad (2)$$

where $e_k \in \Re^3$ are the right-handed basis vectors, with $e_3 = [0, 0, 1]^T$ representing the down-direction and e_1, e_2 the other two canonical directions, $m_i > 0$ is the mass, $x_i \in \Re^3$ is the Cartesian position w.r.t. the inertial frame, $R_i \in SO(3)$ describes the rotation of the body frame w.r.t. the inertial frame, $\lambda_i \in \Re$ is the thrust-force, $w_i := [w_i^1, w_i^2, w_i^3]^T \in \Re^3$ is the body frame's angular rate relative to the inertial frame represented in the body frame, g is the gravitational constant, δ_i is the disturbance (e.g. aerodynamics force Hua et al. (2009)), and $S(\star) : \Re^3 \rightarrow so(3)$ is the skew-symmetric operator defined s.t. for $a, b \in \Re^3$, $S(a)b = a \times b$. Here, without loss of generality, we assume $\lambda_i \in \Re$ is fixed to the body frame $-e_3$ direction (i.e., along $-R_i e_3$ in (1)). We also assume δ_i is negligible, although exponential stability of our proposed control (see Theorems 1 and 2) will guarantee ultimate boundedness against any bounded δ_i. The control inputs are λ_i, w_i^1, w_i^2, which will be defined via backstepping in (22); w_i^3 turns out not necessary (see (22)) and can be conveniently set, e.g., to be $w_i^3 = 0$ to stabilize the yaw motion of TPVs.

We also assume that communication (or sensing) topology among N TPVs is constrained by a time-invariant directed information graph $G := \{\mathcal{V}, \mathcal{E}\}$, where the node set $\mathcal{V} := \{v_1, v_2, \ldots, v_N\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ respectively specify the N TPVs and the (directed) information flow among them, with the self-joining edges excluded (i.e., $(v_i, v_i) \notin \mathcal{E}$). If we assign a weight $w_{ij} > 0$ on each $(v_i, v_j) \in \mathcal{E}$, the graph Laplacian matrix $\mathcal{L} \in \Re^{N\times N}$ of G is defined with its ijth component $\mathcal{L}_{ij}$ given by: $\mathcal{L}_{ij} := -w_{ij}$ if $i \neq j$ and $(v_i, v_j) \in \mathcal{E}$; $\mathcal{L}_{ij} := 0$ if $i \neq j$ and $(v_i, v_j) \notin \mathcal{E}$; and $\mathcal{L}_{ij} := \sum_{k\in\mathcal{N}_i} w_{ik}$ if $i = j$, where $\mathcal{N}_i := \{j|(v_i, v_j) \in \mathcal{E}\}$ is the set of neighbors of the ith TPV. It is then well-known that Ren et al. (2007): (1) all the eigenvalues of $\mathcal{L}$ have non-negative real-part; (2) $\mathbf{1}_N := [1, 1, \ldots, 1]^T \in \Re^N$ is an eigenvector with zero eigenvalue, i.e. $\mathcal{L}\mathbf{1}_N = 0$; and (3) if G is strongly connected, this zero eigenvalue is simple.

Similar to Lee and Spong (2007), in this paper, we particularly consider the case where G is balanced, that is, for each node, its in-degree $in_i(G) := \mathcal{L}_{ii}$ is the same as its out-degree $out_i(G) := -\sum_{j=1,j\neq i}^N \mathcal{L}_{ji}$, $\forall i = 1, 2, \ldots, N$. We then have the following facts about $\mathcal{L}$ (Lee & Spong, 2007; Olfati-Saber & Murray, 2004): (1) the column sums of $\mathcal{L}$ are also zero with $\mathbf{1}_N^T\mathcal{L} = 0$; and (2) if G is strongly-connected, $\mathcal{L} + \mathcal{L}^T$ is a valid Laplacian and has a simple eigenvalue at zero with eigenvector $\mathbf{1}_N$.

The main goal of this paper is to achieve the following two control objectives in E(3) for the N TPVs (1)–(2) on a strongly-connected balanced graph G[3]: (1) *flocking* (Lee & Spong, 2007): $\|x_i - x_j\| \rightarrow 0$ with $\dot{x}_i \rightarrow \bar{v}$ $\forall i, j \in 1, 2, \ldots, N$, where $\bar{v} \in \Re^3$ is an unspecified constant terminal velocity for every TPV; and (2) *formation-centroid control*[4]: $\|x_i - x_j\| \rightarrow 0$ with $(\dot{z}_1, z_1) \rightarrow (\dot{z}_d^1, z_d^1)$, where z_1 is the centroid position as defined by $z_1 := \sum_{i=1}^N (m_i/m_L)x_i \in \Re^3$ with $m_L := \sum_{i=1}^N m_i$ and $z_d^1(t) \in \Re^3$ is a (smooth) desired centroid trajectory.

The key challenge to achieve these objectives is the TPVs' under-actuation in E(3), which renders many consensus/flocking results inapplicable (e.g., Lee and Spong (2007); Olfati-Saber and Murray (2004); Ren (2007); Ren et al. (2007); Su et al. (2009); Tanner et al. (2007); Xiao and Wang (2008)). To address this challenge, in this paper, we extend the passive decomposition (Lee, 2010; Lee & Li, in press, 2007; Lee & Spong, 2007) to explicitly incorporate the topology of G, so that we can design a backstepping control, which can not only overcome this TPVs' under-actuation, but also be distributed over a balanced graph G.

3. Distributed backstepping flocking control of multiple TPVs

Following Lee and Spong (2007), we design the desired control ν_i for the ith TPV (1) s.t.

$$\lambda_i R_i e_3 = \underbrace{b\sum_{j\in\mathcal{N}_i} w_{ij}\dot{e}_{ij} + k\sum_{j\in\mathcal{N}_i} w_{ij}e_{ij} + m_i g e_3}_{=:\nu_i\in\Re^3} + \nu_{ei} \quad (3)$$

[2] Similar to Hua et al. (2009), we consider this "mixed" TPV, since: (1) some commercial TPVs only accept thrust-force and angular-rates (e.g., Asctec Hummingbird®); and (2) we can design angular-torques to duplicate such desired angular-rates for fully-actuated/passive attitude dynamics (Sepulchre et al., 1997).

[3] Due to symmetry of (1)–(2) in E(3), constant offsets $o_{ij} \in \Re^3$ can be incorporated by replacing x_i and z_1^d in (3) (in e_{ij}) and (24) with $\bar{x}_i := x_i - o_i$ and $\bar{z}_1^d := z_1^d - \sum_{l=1}^N (m_l/m_L)o_l$, resulting in $x_i - x_j \rightarrow o_{ij} := o_i - o_j$ and $z_1 \rightarrow z_1^d$. For this, we assume all o_l is known to each TPV.

[4] Although this may also be written as consensus control (Ren, 2007; Su et al., 2009) (i.e., $(\dot{x}_i, x_i) \rightarrow (\dot{z}_1^d, z_1^d)$), here, we adopt formation-centroid control, since (1) in some applications, it is more convenient to design/control formation-centroid behaviors individually; and (2) it can describe some collective behaviors more naturally (e.g., formation-shape/centroid-velocity control by $\|x_i - x_j\| \rightarrow 0$ and $\dot{z}_1 \rightarrow \xi$, with $\xi(t) \in \Re^3$ being a target centroid velocity: see also Section 4).

where $b, k > 0$ are the gains, $w_{ij} > 0$ is the weight on $(v_i, v_j) \in \mathcal{E}$, $e_{ij} := x_i - x_j$, and $\mathcal{N}_i$ is the set of neighbors of ith TPV on the graph G. Here, ν_{ei} is the control-generation error: if $\nu_{ei} = 0$, the dynamics (1) will reduce to that of Lee and Spong (2007) and the flocking be achieved. This $\nu_{ei} = 0$, yet, cannot be assumed a prior since the TPV's Cartesian dynamics (1) is under-actuated. Backstepping law will be designed later (and decoded to the control inputs λ_i, w_i in (22)) to address this issue of under-actuation.

With this ν_i and ν_{ei}, we can then write the ith TPV's closed-loop dynamics as

$$m_i\ddot{x}_i + b\sum_{j\in\mathcal{N}_i} w_{ij}\dot{e}_{ij} + k\sum_{j\in\mathcal{N}_i} w_{ij}e_{ij} = -\nu_{ei}$$

and, stacking them up, we can cast the closed-loop dynamics of the N TPVs into the product form s.t.

$$M\ddot{x} + bL\dot{x} + kLx = -\nu_e \tag{4}$$

where $x := [x_1; x_2; \dots; x_N] \in \Re^{3N}$ and $\nu_e := [\nu_{e1}; \nu_{e2}; \dots, \nu_{eN}] \in \Re^{3N}$ (with $[x; y] := [x^T, y^T]^T$, for $x \in \Re^r, y \in \Re^s$); $M := \mathcal{M} \otimes I_3 \in \Re^{3N\times 3N}$ with $\mathcal{M} := \text{diag}[m_1, m_2, \dots, m_N] \in \Re^{N\times N}$, and $\otimes$ and I_3 respectively being Kronecker product (Brewer, 1978) and the 3×3 identity matrix; and $L := \mathcal{L} \otimes I_3 \in \Re^{3N\times 3N}$ with $\mathcal{L} \in \Re^{N\times N}$ being the graph Laplacian of G. Since G is balanced, yet, directed, $\mathcal{L}$ and L are in general not symmetric.

Now, we extend the passive decomposition of Lee and Spong (2007) by explicitly incorporating the topology of G into it. For this, let us define the following transformations:

$$z := Sx, \qquad \eta := S^{-T}\nu_e \tag{5}$$

with $z =: [z_1; z_e]$, $\eta =: [\eta_1; \eta_e]$ and

$$S := \begin{bmatrix} \mathbf{1}_N^T\mathcal{M}/m_L \\ \Omega_\perp \end{bmatrix} \otimes I_3, \qquad S^{-1} = \begin{bmatrix} \mathbf{1}_N & \Delta_\perp \end{bmatrix} \otimes I_3 \tag{6}$$

where $z_1, \eta_1 \in \Re^3$, $z_e, \eta_e \in \Re^{3(N-1)}$,

$$\Omega_\perp := \begin{bmatrix} \mathcal{L}_1 & l_2 \end{bmatrix} \in \Re^{(N-1)\times N}$$

is the upper-partition of the Laplacian $\mathcal{L} \in \Re^{N\times N}$ s.t.

$$\mathcal{L} =: \begin{bmatrix} \mathcal{L}_1 & l_2 \\ l_3^T & l_4 \end{bmatrix}$$

with $\mathcal{L}_1 \in \Re^{(N-1)\times(N-1)}$, $l_2, l_3 \in \Re^{N-1}$ and $l_4 \in \Re$, and

$$\Delta_\perp := \mathcal{M}^{-1}\Omega_\perp^T(\Omega_\perp\mathcal{M}^{-1}\Omega_\perp^T)^{-1}$$

which identifies the space orthogonal to $\mathbf{1}_N$ w.r.t. the metric $\mathcal{M}$ (i.e., $\mathbf{1}^T\mathcal{M}\Delta_\perp = 0$ from $\Omega_\perp\mathbf{1}_N = 0$) (Lee, 2010).

Here, $\Omega_\perp$ is of full row-rank, $N-1$, since, with $\mathbf{1}^T\mathcal{L} = 0$ (from G being balanced), we have

$$\begin{bmatrix} I_{N-1} & \mathbf{0}_{N-1} \\ \mathbf{1}_{N-1}^T & 1 \end{bmatrix}\begin{bmatrix} \mathcal{L}_1 & l_2 \\ l_3^T & l_4 \end{bmatrix} = \begin{bmatrix} \mathcal{L}_1 & l_2 \\ \mathbf{0}_{N-1}^T & 0 \end{bmatrix}$$

where the most left matrix is full-rank, while $\mathcal{L}$ has rank $N-1$ (from G being strongly-connected), thus, the most right matrix (i.e., $\Omega_\perp$) should have the rank $N-1$. Subsequently, S is also full-rank (thus, invertible), since $\mathbf{1}_N^T M/m_L$ in (6) is not contained within the row-space of $\Omega_\perp$: if so, there should exist a non-zero $y \in \Re^N$ s.t. $y^T\Omega_\perp = \mathbf{1}^T M/m_L$, which, yet, is impossible, since, post-multiplying it by $\mathbf{1}_N$, we have $0 = y^T\Omega_\perp\mathbf{1}_N = \mathbf{1}_N^T M\mathbf{1}_N/m_L = 1$. We can also directly check $SS^{-1} = I_{3N}$ for (6) using the fact that $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$ for compatible A, B, C, D (Brewer, 1978).

From (5)–(6), we can then see that: (1) $z_1 = \sum_{i=1}^N (m_i/m_L)x_i \in \Re^3$ is indeed the centroid position (with $\mathbf{1}_N^T\mathcal{M}/m_L = [m_1, m_2, \dots, m_N]/m_L$); and (2) $z_e = (\Omega_\perp \otimes I_3)x \in \Re^{3(N-1)}$ specifies the formation shape among N TPVs (e.g., $z_e = [x_1 - x_4; x_2 - x_1; x_3 - x_2]$ for four agents on a cyclic graph G), with $z_e \to 0$ implying $\|x_i - x_j\| \to 0$, which is because, from $\Omega_\perp$ being full row-rank, the null-space of $(\Omega_\perp \otimes I_3)$ is given by $\mathbf{1}_N \otimes c$ for an arbitrary $c \in \Re^3$ (i.e., $z_e = 0 \Leftrightarrow x_i = x_j = c$).

Using (5)–(6), we can then decompose the product dynamics (4) of N TPVs into

$$m_L\ddot{z}_1 = -\eta_1 \tag{7}$$

$$\bar{M}\ddot{z}_e + b\bar{L}\dot{z}_e + k\bar{L}z_e = -\eta_e \tag{8}$$

where

$$S^{-T}MS^{-1} =: \text{diag}[m_L \otimes I_3, \bar{M}]$$

with $\bar{M} = \Delta_\perp^T\mathcal{M}\Delta_\perp \otimes I_3 \in \Re^{3(N-1)\times 3(N-1)}$ and $\mathbf{1}_N^T\mathcal{M}\Delta_\perp = 0$ (from $\Omega_\perp\mathbf{1}_N = 0$); and

$$S^{-T}LS^{-1} =: \text{diag}[\mathbf{0}_3, \bar{L}] \tag{9}$$

with $\bar{L} = \Delta_\perp^T\mathcal{L}\Delta_\perp \otimes I_3 \in \Re^{3(N-1)\times 3(N-1)}$ and $\mathbf{1}_N^T\mathcal{L} = 0$.

Here, with G being balanced and strongly-connected, $\bar{L}_{\text{sym}} := (\bar{L} + \bar{L}^T)/2 \succ 0$ (i.e. positive-definite). To see this, note from (9) that $S^{-T}L_{\text{sym}}S^{-1} = \text{diag}[\mathbf{0}_3, \bar{L}_{\text{sym}}]$, where $L_{\text{sym}} := (L + L^T)/2 = \mathcal{L}_{\text{sym}} \otimes I_3$, with $\mathcal{L}_{\text{sym}} := (\mathcal{L} + \mathcal{L}^T)/2$. Thus, from the spectral property of $\mathcal{L}_{\text{sym}}$ in Section 2, and due to the facts that the eigenvalues of L_{sym} are the products of those of $\mathcal{L}_{\text{sym}}$ and I_3 (Brewer, 1978) and the congruence transform preserves the signs of the eigenvalues (Lee & Spong, 2007), this L_{sym} possesses 3 zero eigenvalues, and $3(N-1)$ strictly-positive eigenvalues, which correspond to the strictly-positive $3(N-1)$ eigenvalues of $\bar{L}_{\text{sym}}$, implying that $\bar{L}_{\text{sym}} \succ 0$.

For the z_e-dynamics (8), we can then show that

$$\frac{dV_1}{dt} = -\begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix}^T \underbrace{\begin{bmatrix} b\bar{L}_{\text{sym}} - \epsilon\bar{M} & \dfrac{k - \epsilon b}{2}\bar{L}_{\text{skew}} \\ -\dfrac{k-\epsilon b}{2}\bar{L}_{\text{skew}} & \epsilon k\bar{L}_{\text{sym}} \end{bmatrix}}_{=:Q} \begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix} - (\dot{z}_e + \epsilon z_e)^T\eta_e \tag{10}$$

with $\epsilon > 0$ (to be designed below), where

$$V_1 := \frac{1}{2}\begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix}^T \underbrace{\begin{bmatrix} \bar{M} & \epsilon\bar{M} \\ \epsilon\bar{M} & (k + \epsilon b)\bar{L}_{\text{sym}} \end{bmatrix}}_{=:P} \begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix} \tag{11}$$

and $\bar{L}_{\text{skew}} := (\bar{L} - \bar{L}^T)/2$. Thus, if the last term of (10) is absent (e.g., $\nu_{ei} = 0$), and $P \succ 0$ and $Q \succ 0$, we would achieve $(\dot{z}_e, z_e) \to 0$.

Now, to address the last term of (10), which is induced by the TPV's under-actuation, we design a backstepping control. For this, we augment V_1 s.t.

$$V_2 := V_1 + \frac{1}{2}\eta_e^T\Gamma^{-1}\eta_e = V_1 + \frac{1}{2}\eta_e^T\Gamma_{\text{sym}}^{-1}\eta_e \tag{12}$$

where $\Gamma \in \Re^{3(N-1)\times 3(N-1)}$ will be defined below to be invertible and $\Gamma_{\text{sym}}^{-1} = (\Gamma^{-1} + \Gamma^{-T})/2 \succ 0$. Using this V_2 and (8), we can then have

$$\frac{dV_2}{dt} = -\begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix}^T Q \begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix} - \frac{1}{2}(\dot{z}_e + \epsilon z_e)^T\eta_e - \frac{1}{2}\eta_e^T(\dot{z}_e + \epsilon z_e) + \frac{1}{2}\eta_e^T\Gamma^{-1}\dot{\eta}_e + \frac{1}{2}\dot{\eta}_e^T\Gamma^{-1}\eta_e$$

which suggests the following *target* update law for η_e:

$$\dot{\eta}_e = \Gamma(\dot{z}_e + \epsilon z_e) - \alpha\eta_e \tag{13}$$

with $\alpha > 0$. Adopting this (13), we can then obtain

$$\frac{dV_2}{dt} = -\zeta^T Q_e\zeta$$

where $\zeta := [\dot{z}_e; z_e; \eta_e]$ and

$$Q_e := \begin{bmatrix} b\bar{L}_{\rm sym} - \epsilon\bar{M} & \frac{k-\epsilon b}{2}\bar{L}_{\rm skew} & \frac{I-\Gamma^T\Gamma^{-1}}{4} \\ -\frac{k-\epsilon b}{2}\bar{L}_{\rm skew} & \epsilon k\bar{L}_{\rm sym} & \frac{\epsilon[I-\Gamma^T\Gamma^{-1}]}{4} \\ \frac{I-\Gamma^{-T}\Gamma}{4} & \frac{\epsilon[I-\Gamma^{-T}\Gamma]}{4} & \alpha\Gamma_{\rm sym}^{-1} \end{bmatrix}. \quad (14)$$

Following Horn and Johnson (1985) and Lee and Spong (2007, Theorem 1), this Q_e in (14) will be positive-definite, if

$$b\bar{L}_{\rm sym} - \epsilon\bar{M} \succ 0, \quad \epsilon = k/b \quad (15)$$

and

$$b\cdot\alpha\cdot\Gamma_{\rm sym}^{-1} \succ \frac{I-\Gamma^{-1}\Gamma}{4}\left[\left(\bar{L}_{\rm sym} - \frac{k}{b^2}\bar{M}\right)^{-1} + \bar{L}_{\rm sym}^{-1}\right]\frac{I-\Gamma^T\Gamma^{-1}}{4} \quad (16)$$

which implies that, given $\Gamma, k > 0$, there always exist (large enough) $b > 0$ and $\alpha > 0$ to make $Q_e \succ 0$. Moreover, since (15) also enforces $P \succ 0$ for V_1 in (11) (i.e., $P \succ 0 \Leftrightarrow (k+\epsilon b)\bar{L}_{\rm sym} \succ \epsilon^2\bar{M}$ (Horn & Johnson, 1985)), satisfying (15)–(16) will imply $\dot{z}_e$, z_e and η_e all converge to zero *exponentially*.

The target update law (13), yet, appears centralized (e.g., requires $\dot{z}_e, z_e, \eta_e$ information). Here, using the decomposition (5)–(6), that is made "compatible" to G by embedding the topology of G into it, we show that the target law (13) is in fact decentralizable over G. For this, consider the following distributed update law for ν_{ei}:

$$\dot{\nu}_{ei} = \gamma\sum_{j\in\mathcal{N}_i} w_{ij}(\dot{e}_{ij} + \epsilon e_{ij}) - \alpha\nu_{ei} \quad (17)$$

with $\gamma > 0$; or equivalently, in the product form,

$$\dot{\nu}_e = \gamma L(\dot{x} + \epsilon x) - \alpha\nu_e \quad (18)$$

which is distributed on G, as manifested by L therein.

Recall also from (5) that

$$\eta_e = (\Delta_\perp \otimes I_3)^T\nu_e.$$

We can then transform the distributed update law (18) into $\dot{\eta}_e$ s.t.

$$\dot{\eta}_e = (\Delta_\perp \otimes I_3)^T\dot{\nu}_e = \gamma(\Delta_\perp \otimes I_3)^T L(\dot{x} + \epsilon x) - \alpha\eta_e$$

which will match the target update law (13), if

$$\begin{aligned}\Gamma(\dot{z}_e + \epsilon z_e) &= \gamma(\Delta_\perp \otimes I_3)^T L(\dot{x} + \epsilon x) \\ &= \gamma(\Delta_\perp \otimes I_3)^T L(\Delta_\perp \otimes I_3)(\dot{z}_e + \epsilon z_e)\end{aligned}$$

where we use the fact that, from (5) to (6), $\dot{x} = (\mathbf{1}_N \otimes I_3)\dot{z}_1 + (\Delta_\perp \otimes I_3)\dot{z}_e$, with $L(\mathbf{1}_N \otimes I_3) = (\mathcal{L}\mathbf{1}_N) \otimes I_3 = 0$ (similar also hold for x). This shows that the distributed update law (18) will duplicate the target update law (13), if we choose Γ s.t.

$$\Gamma := \gamma(\Delta_\perp \otimes I)^T L(\Delta_\perp \otimes I) = \gamma\bar{L} \quad (19)$$

which can also be used for (12), since: (1) Γ is invertible, because so is $\bar{L}$ (i.e., from (9), $\mathrm{rank}(\bar{L}) = 3(N-1)$); and (2) $\Gamma_{\rm sym}^{-1} \succ 0$ from $\Gamma_{\rm sym} = \gamma\bar{L}_{\rm sym} \succ 0$ (see the paragraph after (9)).

So far we have established exponential stability of z_e-dynamics (8). What remains to show now is the stability of z_1-dynamics (7). For this, note from (5) that $\eta_1 = (\mathbf{1}_N \otimes I_3)^T\nu_e$. With the distributed law (18), we then have

$$\dot{\eta}_1 = (\mathbf{1}_N \otimes I_3)^T[L(\dot{x} + \epsilon x) - \alpha\nu_e] = -\alpha\eta_1$$

with $(\mathbf{1}_N \otimes I_3)^T L = \mathbf{1}_N^T\mathcal{L} \otimes I_3 = 0$. This implies that η_1 in (7) is by itself exponentially decaying, and the z_1-dynamics (7) is stable with bounded $\ddot{z}_1$ and $\dot{z}_1$, although z_1 is generally not so. Integrating (7) with $\eta_1(t) = \eta_1(0)e^{-\alpha t}$, we can also compute the (constant) terminal centroid velocity s.t.

$$\dot{z}_1(t) = \dot{z}_1(0) + \frac{\eta_1(0)}{\alpha m_L}[e^{-\alpha t} - 1] \to \dot{z}_1(0) - \frac{\eta_1(0)}{\alpha m_L} \quad (20)$$

which shows that: (1) in contrast to Lee and Spong (2007), due to the TPV's under-actuation, centroid velocity $\dot{z}_1$ is in general not invariant (i.e., $\dot{z}_1(t) \neq \dot{z}_1(0)$); (2) if $\eta_1(0) = 0$, $\dot{z}_1(t) = \dot{z}_1(0)$ (i.e., invariant $\dot{z}_1$); and (3) the larger m_L and α are, the closer $\dot{z}_1(t)$ is to $\dot{z}_1(0)$ (i.e,. almost invariant $\dot{z}_1$). With $\dot{z}_e \to 0$ (i.e. $\dot{x}_i - \dot{x}_j \to 0$), we also have $\dot{z}_1 = \sum_{i=k}^N (m_k/m_L)\dot{x}_k \to \dot{x}_i$, implying that (20) also specifies the terminal velocity for every TPV, i.e., $\dot{x}_i \to \dot{z}_1(0) - \eta_1(0)/(\alpha m_L)$ $\forall i = 1, 2, \ldots, N$. Moreover, with $\eta_1 \to 0$ and $\eta_e \to 0$ (with $P, Q_e \succ 0$ for (14)) both exponentially, $\nu_{ei} \to 0$ exponentially as well due to (5).

The distributed backstepping control law, (3) and (17), then needs to be decoded into the (real) control inputs λ_i, w_i^1, w_i^2 for each TPV. For this, using (17) with $\nu_{ei} = \lambda_i R_i e_3 - \nu_i$ (from (3)) and $\dot{\nu}_{ei} = (d\lambda_i/dt)R_i e_3 + \lambda_i R_i S(w_i)e_3 - \dot{\nu}_i$ (using (2)), we can obtain

$$\begin{aligned}&[(\dot{\lambda}_i + \alpha\lambda_i)R_i + \lambda_i R_i S(w_i)]e_3 \\ &\quad = \dot{\nu}_i + \alpha\nu_i + \gamma\sum_{j\in\mathcal{N}_i} w_{ij}(\dot{e}_{ij} + \epsilon e_{ij}) =: \bar{\nu}_i \end{aligned} \quad (21)$$

from which we can extract the control law for each TPV:

$$\begin{bmatrix}\lambda_i w_i^2 & -\lambda_i w_i^1 & \dot{\lambda}_i + \alpha\lambda_i\end{bmatrix}^T = R_i^T\bar{\nu}_i \quad (22)$$

where, to compute $\dot{\nu}_i$ in (21) without utilizing (usually inaccessible) $\ddot{x}_i$, we also use $\dot{\nu}_i = -b\sum_{j\in\mathcal{N}_i} w_{ij}[(\lambda_i/m_i)R_i - (\lambda_j/m_j)R_j]e_3 + k\sum_{j\in\mathcal{N}_i} w_{ij}\dot{e}_{ij}$, which can be obtained by differentiating ν_i in (3) with (1).

The detailed expression of $R_i^T\bar{\nu}_i$ in (22) with the offsets $o_{ij} := o_i - o_j$ (see the footnote 2) and $e_{ij} := x_i - x_j$ is then given by

$$\begin{aligned}R_i^T\bar{\nu}_i &= -b\sum_{j\in\mathcal{N}_i} w_{ij}\left(\frac{\lambda_i}{m_i}e_3 - \frac{\lambda_j}{m_j}R_i^T R_j e_3\right) + \alpha m_i g R_i^T e_3 \\ &\quad + (\alpha b + k + \gamma)\sum_{j\in\mathcal{N}_i} w_{ij}R_i^T\dot{e}_{ij} \\ &\quad + (\alpha k + \epsilon\gamma)\sum_{j\in\mathcal{N}_i} w_{ij}[R_i^T e_{ij} - R_i^T o_{ij}]\end{aligned} \quad (23)$$

which implies that: (1) the control law (22) is distributed over G, with each agent required to have access to their neighbors' body e_3-direction, relative velocity and relative distance, all measured from its own body-frame (i.e., $R_i^T R_j e_3$, $R_i^T\dot{e}_{ij}$, $R_i^T e_{ij}$), λ_j/m_j (e.g., via communication or input-observer), and its own attitude R_i w.r.t. the inertial frame (e.g., via common landmarks) to implement $gR_i^T e_3$ and $R_i^T o_{ij}$ in (23); and (2) with the third row of (22), similar to the case of dynamic extension (Sepulchre et al., 1997), λ_i becomes a state, thus, each agent can compute the angular inputs s.t. $w_i^1(t) = -(R_i^T\bar{\nu}_i)_2/\lambda_i(t)$ and $w_i^2(t) = (R_i^T\bar{\nu}_i)_1/\lambda_i(t)$ from (22) assuming that $\lambda_i(t) \neq 0$ (or, $\ddot{x}_i \neq ge_3$ from (1), i.e., no free-fall (Frazzoli et al., 2000; Hua et al., 2009)) while also updating $\lambda_i(t)$ via the third row of (22) without any algebraic-loop. Note that w_i^3 is not needed for (22), thus, can be set by $w_i^3 = 0$ to stabilize the TPV's yaw motion.

The conditions (15)–(16) for the control gains $b, k, \epsilon, \alpha, \gamma$ can also always be satisfied by choosing b, α large enough, given k, γ. If G is undirected, these conditions (15)–(16) can be further relaxed, that is, since both $\bar{L}$ and $\Gamma = \gamma\bar{L}$ are symmetric, all the off block-diagonal terms in (14) vanish, thus, the conditions (15)–(16) simply boil down to the (single) condition $b\bar{L} - \epsilon\bar{M} \succ 0$, under which we can choose *any* $b, k, \epsilon, \alpha, \gamma > 0$.

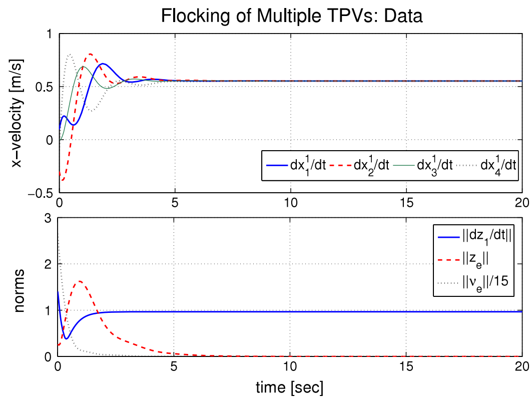

Fig. 1. Flocking of four TPVs.

Being exponentially stable, $(z_e, \eta_e) \to 0$ is also robust (i.e., $\|z_e\|$, $\|\eta_e\|$ ultimate-bounded) against any bounded disturbance (e.g., aerodynamics force δ_i in (1); model uncertainty effect). Parametric uncertainty can also be incorporated into the matrix inequalities $P \succ 0, Q_e \succ 0$ for (11) and (14). Instead of $\epsilon = b/k$ in (15), we may also choose ϵ by directly solving $P \succ 0, Q_e \succ 0$, which defines linear matrix inequality (LMI) w.r.t. ϵ. We now summarize our results in the following Theorem 1.

Theorem 1. *Consider N TPVs (1)–(2) on a balanced and strongly-connected graph G under the distributed backstepping flocking control (22)–(23), with its gains $b, k, \epsilon, \alpha, \gamma$ chosen according to (15)–(16) (or only the first item of (15) if G is undirected) and $\delta_i = 0$ for (1). Then, we have: $\forall i, j = 1, 2, \ldots, N$,*

(1) $\|\dot{x}_i - \dot{x}_j\| \to 0$ *and* $\|x_i - x_j\| \to 0$ *exponentially;*
(2) $\dot{x}_i \to \sum_{k=1}^N \frac{m_k}{m_L}\dot{x}_k(0) - \frac{\eta_1(0)}{\alpha m_L}$ *exponentially;*
(3) $\nu_{ei} \to 0$ *exponentially; and*
(4) $\dot{z}_1(t) = \dot{z}_1(0) + \frac{\eta_1(0)}{\alpha m_L}[e^{-\alpha t} - 1]$.

We performed a simulation using four TPVs (1)–(2) on a cyclic graph G with $w_{ij} = 1$ and $in_i(G) = out_i(G) = 1\ \forall i = 1, 2, 3, 4$. We also choose $(m_1, m_2, m_3, m_4) = (2, 1, 2.5, 2)$ [kg]; inter-TPV offsets with $o_1 = [1; 1; 0]$ [m], $o_2 = [-1, 1, 0]$ [m], $o_3 = [-1, -1, 0]$ [m], $o_4 = [-1, -1, 0]$ [m] (see footnote 2); and $b = 3.5$ [Ns/m], $k = 2.5$ [N/m], $\gamma = 3.5$ [Ns/m] and $\alpha = 2.5$ [1/s] according to (15)–(16). Some representative data are shown in Fig. 1, where we can see that: (1) desired formation shape is achieved (i.e., $\|z_e\| \to 0 \Leftrightarrow x_i - x_j \to o_{ij}$); (2) each TPV's e_1-velocity $\dot{x}_i^1$ and their centroid velocity $\dot{z}_1$ converge to their respective constant terminal values; and (3) the control generation-error ν_e also decays to zero.

4. Distributed backstepping formation-centroid control of multiple TPVs

Now, suppose that we not only want to achieve the desired formation shape among the N TPVs, but also to control their centroid position z_1 to track a certain trajectory $z_1^d(t) \in \Re^3$ on a balanced and strongly-connected graph G. Compared to the flocking of Section 3, this formation-centroid control is more often desired in practice (e.g., domain coverage). For this, we modify the desired control (3) s.t.

$$\lambda_i R_i e_3 = \nu_i + \underbrace{m_i[\lambda_b(\dot{x}_i - \dot{z}_1^d) + \lambda_k(x_i - z_1^d) - \ddot{z}_1^d]}_{=:\nu_{Li}\in\Re^3} + \nu_{ei} \tag{24}$$

where $\nu_i \in \Re^3$ is the flocking control defined in (3), $\nu_{Li} \in \Re^3$ is the newly-added centroid tracking control with the gains $\lambda_b, \lambda_k > 0$, and $\nu_{ei} \in \Re^3$ is the control-generation error. Here, we assume that $(z_1^d, \dot{z}_1^d, \ddot{z}_1^d, \dddot{z}_1^d)$ is available to all TPVs (similar to (Aguiar & Hespanha, 2007; Hua et al., 2009)), yet, not $(z_1, \dot{z}_1)$ which requires all the TPVs' $(x_i, \dot{x}_i)$.

Then, similar to Section 3, stacking up the N TPV dynamics, we can obtain their product dynamics s.t.

$$M\ddot{x} + bL\dot{x} + kLx + \nu_L = -\nu_e$$

where $\nu_L := [\nu_{L1}; \nu_{L2}; \ldots \nu_{LN}] \in \Re^{3N}$, and, applying the transformation (5)–(6), we can also achieve

$$m_L\ddot{z}_1 + (\mathbf{1}_N \otimes I_3)^T \nu_L = -\eta_1 \tag{25}$$

$$\bar{M}\ddot{z}_e + b\bar{L}\dot{z}_e + k\bar{L}z_e + (\Delta_\perp \otimes I_3)^T \nu_L = -\eta_e \tag{26}$$

similar to (7)–(8). Here, the term $(\mathbf{1}_N \otimes I_3)^T \nu_L$ in (25) produces the centroid tracking action, that is, $(\mathbf{1}_N \otimes I_3)^T \nu_L = \sum_{i=1}^N m_i[\lambda_b(\dot{x}_i - \dot{z}_1^d) + \lambda_k(x_i - z_1^d) - \ddot{z}_1^d] = m_L[\lambda_b(\dot{z}_1 - \dot{z}_1^d) + \lambda_k(z_1 - z_1^d) - \ddot{z}_1^d]$, with which we can rewrite (25) as

$$m_L[(\ddot{z}_1 - \ddot{z}_1^d) + \lambda_b(\dot{z}_1 - \dot{z}_1^d) + \lambda_k(z_1 - z_1^d)] = -\eta_1 \tag{27}$$

implying that, if $\eta_1 \to 0$, $(\dot{z}_1 - \dot{z}_1^d, z_1 - z_1^d) \to 0$.

Let us also consider the z_e-dynamics (26). A rather surprising fact here is that the term $(\Delta_\perp \otimes I)^T \nu_L$ in (26) does not at all perturb the convergence of $\|z_e\| \to 0$, but rather enhances its robustness, all the while still allowing us to use the *same* form of the previous flocking backstepping control (17) (i.e., (13)). To see this, first note from (24) that we can write $\nu_L = M(\lambda_b\dot{x} + \lambda_k x) - M\mathbf{1}_N \otimes (\ddot{z}_1^d + \lambda_b\dot{z}_1^d + \lambda_k z_1^d)$, with which we can reduce $(\Delta_\perp \otimes I_3)^T \nu_L$ in (26) into

$$\begin{aligned}(\Delta_\perp \otimes I_3)^T \nu_L &= (\mathcal{D}\Omega_\perp \otimes I_3)(\lambda_b\dot{x} + \lambda_k x)\\ &= (\mathcal{D} \otimes I_3)(\lambda_b\dot{z}_e + \lambda_k z_e)\end{aligned}$$

where $\mathcal{D} := (\Omega_\perp M^{-1}\Omega_\perp^T)^{-1} \succ 0$ and we use the fact that $(\Delta_\perp \otimes I_3)^T M\mathbf{1}_N \otimes y = (\mathcal{D}\Omega_\perp \otimes I_3)(\mathbf{1}_N \otimes y) = (\mathcal{D}\Omega_\perp\mathbf{1}_N) \otimes y = 0$, $\forall y \in \Re^3$, from the properties of Kronecker product (Brewer, 1978) and $\Omega_\perp\mathbf{1}_N = 0$.

We can then rewrite (26) s.t.

$$\bar{M}\ddot{z}_e + (b\bar{L} + \lambda_b D)\dot{z}_e + (k\bar{L} + \lambda_k D)z_e = -\eta_e \tag{28}$$

where $D := \mathcal{D} \otimes I_3 \succ 0$. With this D being symmetric, we can further obtain the same relation (10) for (28), with only P, Q in (10)–(11) replaced by

$$P' := P + \begin{bmatrix} 0 & 0 \\ 0 & (\lambda_k + \epsilon\lambda_b)D \end{bmatrix}, \qquad Q' := Q + \begin{bmatrix} \lambda_b D & 0 \\ 0 & \epsilon\lambda_k D \end{bmatrix}.$$

This means that we can utilize exactly the same distributed backstepping law (17) of Section 3 here (with ν_{ei} given by (24)) to achieve exponential stability of $z_e, \dot{z}_e, \eta_e$ (and η_1), which, in turn, enforces exponential convergence of $(z_1, \dot{z}_1) \to (z_1^d, \dot{z}_1^d)$ as well. This also manifests that the term $(\Delta_\perp \otimes I)^T \nu_L$ in (26) improves robustness of the exponential stability of z_e-dynamics by making P and Q_e for (15)–(16) more positive-definite (i.e., relaxing the conditions (15)–(16)).

Combining (17) and (24) with the offsets o_{ij} in the footnote 2, we can then obtain the same control decoding relation as (22) with $R_i^T\bar{\nu}_i$ there replaced by

$$\begin{aligned}R_i^T\bar{\nu}_i' &:= R_i^T(\bar{\nu}_i + \dot{\nu}_{Li} + \alpha\nu_{Li})\\ &= R_i^T\bar{\nu}_i + \lambda_b(m_i g R_i^T e_3 - \lambda_i e_3) + \alpha m_i\lambda_k R_i^T(\bar{x}_i - \bar{z}_1^d)\\ &\quad + m_i(\lambda_k + \alpha\lambda_b)R_i^T(\dot{x}_i - \dot{z}_1^d)\\ &\quad - m_i R_i^T(\dddot{z}_1^d + (\lambda_b + \alpha)\ddot{z}_1^d)\end{aligned} \tag{29}$$

where $R_i^T\bar{\nu}_i$ is given in (23), and $\bar{x}_i$ and $\bar{z}_1^d$ in the footnote 2. For this, similar to Section 3, we also use $\dot{\nu}_{Li} = -\lambda_b(\lambda_i R_i e_3 - m_i g e_3 + m_i\ddot{z}_1^d) + m_i[\lambda_k(\dot{x}_i - \dot{z}_1^d) - \dddot{z}_1^d]$ to circumvent the usage of $\ddot{x}_i$.

The expression (29) shows that the achieved control is distributable over G, with each agent now required to access its body frame position and velocity (i.e., $R_i^T x_i$, $R_i^T \dot{x}_i$; via, e.g., triangulation w.r.t. common landmarks) and $z_1^d, \dot{z}_1^d, \ddot{z}_1^d, \dddot{z}_1^d$ (e.g., stored before operation), on top of the necessary information for $R_i^T \bar{\nu}_i$ as stated after (23). All the observations made before Theorem 1 also still hold here, due to the similarity of stability analysis and the exponential stability of (z_1, z_e, η_e)-dynamics. The formation control ν_i in (24) also enforces coherency among the agents (e.g., no agents left behind while others flying away (Lee & Li, 2007)), in contrast to the case of simply making each agent track its own trajectory individually.

Note from (27)–(28) that formation and centroid controls do not interfere with each other, except the exponentially-decaying crosstalk η_1, η_e. This means that the formation and centroid behaviors can be designed/controlled separately and simultaneously. This (useful) *formation-centroid decoupling* would not be possible, if the topology of G were not properly incorporated into (5)–(6) (Lee & Li, 2007); or if the TPVs did not cooperate with each other (e.g., only one TPV implements (24)). The following Theorem 2 summarizes our results so far in this Section 4.

Theorem 2. *Consider N TPVs (1)–(2) on a balanced and strongly-connected graph G, under the distributed backstepping formation-centroid control (22) with $R_i^T \bar{\nu}_i$ replaced by $R_i^T \bar{\nu}_i'$ (29), $\lambda_b, \lambda_k > 0$, the gains $b, k, \epsilon, \alpha, \gamma$ chosen according to (15)–(16) (or only the first item of (15) if G is undirected), and $\delta_i = 0$ for (1). Then, we have:*

(1) $(z_e, \dot{z}_e) \to 0$ *exponentially;*
(2) $\nu_{ei} \to 0$ *exponentially* $\forall i = 1, 2, \ldots, N$*; and*
(3) $(z_1 - z_1^d, \dot{z}_1 - \dot{z}_1^d) \to 0$ *exponentially.*

An immediate corollary of Theorem 2 is that, if $\lambda_k = 0$, $\dot{z}_1 \to \dot{z}_1^d$ (but, in general, $z_1 \nrightarrow z_1^d$) and $(z_e, \dot{z}_e, \nu_{ei}) \to 0$ exponentially (i.e., formation-shape/centroid-velocity control - see footnote 3). We may also think of $(\dot{z}_1^d, z_1^d)$ as the virtual leader. However, available results on this virtual leader (e.g., (Ren, 2007; Su et al., 2009)) typically: (1) require full-actuation, thus, not directly applicable to the under-actuated TPVs; and (2) focus mainly on the asymptotic consensus behaviors, thus, not able to establish the (useful) formation-centroid decoupling as we could in (25)–(26). How to combine these virtual leader results with our decomposition-based backstepping approach, particularly when only a portion of all the (under-actuated) agents is informed of $(\dot{z}_1^d, z_1^d)$ and/or when the graph G is time-varying, is a topic for future research.

We performed a simulation using the setting of Fig. 1 with a Lissajous-like trajectory $z_d^1(t) = (r_m \cos(2w_d t + \pi/4), r_m \sin(2w_d t + \pi/4), h_m \cos(w_d t) - 1)$, where $(r_m, h_m) = (2.5, 3.5)$ [m] and $w_d = 0.1\pi$ [rad/s]. We choose $\lambda_b = 6.75$ [Ns/m], $\lambda_k = 6$ [N/m]; and also use $\bar{z}_d^1(t) = z_d^1(t) - \sum_{i=1}^4 (m_i/m_L) o_i$ to incorporate inter-TPV offsets (see footnote 2). Some representative data are presented in Fig. 2, where we can see that: (1) desired formation (i.e., $\|z_e\| \to 0$) and centroid tracking (i.e., $\|z_1 - z_1^d\| \to 0$) are achieved simultaneously; (2) all the TPVs' altitudes (i.e., x_i^3) synchronize, as the desired formation shape is a planar square; and (3) control-generation error ν_{ei} converges to zero for each TPV.

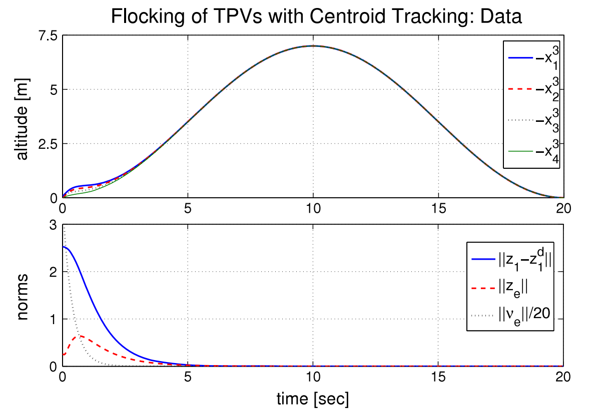

Fig. 2. Formation-centroid control of four TPVs.

5. Summary and future research

In this paper, we propose novel exponentially-stable distributed control frameworks for TPVs on a strongly-connected and balanced graph G. To address the issue of TPVs' under-actuation, we adopt the backstepping technique, while, to distribute the control over G, we extend the passive decomposition of Lee and Spong (2007) to explicitly incorporate the topology of G. Two control objectives are solved: flocking, and flocking with centroid tracking. Simulations are also performed.

Some future research topics include: (1) extension to unbalanced and switching graph G; (2) inclusion of collision/obstacle avoidance; (3) further relaxation of communication/sensing requirement of (23) and (29); (4) real TPVs implementation and robustification; and (5) extension to other classes of under-actuated systems.

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